Technical Change and the Rate of Profit in Classical-Marxian Models of Economic Growth

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Abstract

I study the effect of technical change on the equilibrium profit rate in Classical-Marxian models of economic growth with alternative closures. In each model, capitalists adopt a new technique of production only if it will increase the profit rate given their expectations about the movement of the real wage rate. The accumulation rate depends on a threshold rate of profit, below which capitalists do not invest. I consider three alternative closures: (a) a constant real wage rate (relevant for a labor surplus economy); (b) a constant wage share (relevant for an advanced capitalist economy with strong labor); and (c) a constant unemployment rate (relevant for an advanced capitalist economy with weak labor). For the model of the advanced capitalist economy with strong labor, the profit rate can fall after viable technical change irrespective of capitalists' expectations about the trajectory of the real wage rate after technical change. For models of the labor surplus economy and the advanced capitalist economy with weak labor, the equilibrium rate of profit can fall after viable technical change only if capitalists' choice of technique had been based on an expected fall in the real wage rate after technical change.

JEL Codes: B51; E11; O41.

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1 Introduction

The impact of technical change on the profit rate has been an important question in classical political economy. For David Ricardo, resource constraints would push a capitalist economy

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towards a stationary state of zero growth in the long run, unless technical progress, which increased the productivity or labor and capital, came to the rescue.

Karl Marx, a dialectical thinker par excellence, presented a more contradictory picture of technical progress in capitalism (Marx, 1993). Technical change, Marx argued, is at one and the same time its greatest strength and weakness. While technical makes the capitalist system progressive and open up the possibilities of material progress, it can also push the system towards stagnation by driving the profit rate down. If the profit rate falls below the minimum threshold level necessary to induce capitalists to undertake investment, capital accumulation, the very engine of growth in capitalism, can fall to zero.

Marx's argument about the law of tendential fall in the rate of profit (LTFRP) was simple and straightforward. Technical change is capitalism is characterized by the unrelenting march of mechanization, of labor being replaced by capital. While this will increase labor productivity, it will also reduce capital productivity (the ratio of output and the capital stock). Such a pattern of technical change will impart a tendency to the equilibrium profit rate to fall if the rate of exploitation, or equivalently the profit share in national income, remains relatively stable.

While many generations of scholars and activists debated Marx's LTFRP since the publication of volume 3 of *Capital*, it was Nobuo Okishio's contribution which propelled the literature in a whole new direction. Okishio (1961) argued that technical change occurs in capitalist economies when capitalist firms adopt new techniques of production. But capitalist firms will adopt a new technique of production only when it reduces the cost of production at the prevailing prices and wages. We can call such a new technique of production as (capitalistically) *viable*. Okishio (1961) demonstrated, in a multi-sector model with linear technologies of production, that the equilibrium profit rate will rise, and not fall, with viable technical change if the real wage rate remains constant.

Later scholars have showed that Marx's and Okishio's results can both hold (Laibman, 1982; Foley, 1986; Blecker and Setterfield, 2019; Basu, 2021; Basu and Orellana, 2023). After all, Marx and Okishio worked with different assumptions about the trajectory of real wages after technical change. While Okishio assumed that the real wage rate remains unchanged, Marx had assumed that the wage share remained unchanged. In fact, Marx's assumption implies that the real wage rate has to rise to keep the wage share unchanged (because labor productivity generally rises with technical change). One can go further and show that, in a one-sector model of production, if the growth rate of the real wage rate is above a threshold (the Marx-Okishio threshold), then Marx's result obtains; if it is below the threshold then Okishio's result obtains (Foley, 1986; Basu, 2021, chapter 6.4).

While insightful in many respects, the Marx-Okishio literature on technical change and profitability can be enriched further by paying closer attention to the context provided by the specific structure of the economy. After all a labor surplus developing economy undergoing capitalist development might work with a different logic, compared to a developed capitalist economy, so far as the impact of technical change on profitability is concerned. Such a perspective, predating the Okishio-inspired debates by several debates, seems to have informed the work of Maurice Dobb (Dobb, 1945, chapter iv).

In this paper, I revisit Marx's LTFRP by studying the impact of viable technical change on the equilibrium rate of profit in classical-Marxian model of economic growth. I connect with Maurice Dobb's useful analysis by working with alternative labor market closures of the basic classical-Marxian growth model.[1] My interpretation of these alternative closures maps them into different structural and institutional settings of capitalist economies: a labor surplus developing economy, an advanced capitalist economy with strong labor, and an advanced capitalist economy with weak labor.

The basic model I work with is the classical-Marxian growth model presented in Blecker

[1]This strategy of using alternative closures of classical models of economic growth can also be seen in Kaldor (1961).

and Setterfield (2019, section 2.4.3) and have the following features: (a) Leontief technology of production; (b) choice of technique by capitalist firms based on profit rate comparisons before and after technical change; (c) an accumulation (saving/investment) function with a minimum threshold for positive investment; and (d) a fixed, normal (or desired) capacity utilization in the long run.

To these elements of the model, I add a *novel* feature, drawing on and extending Basu (2010). In the literature on the choice of technique inaugurated by Okishio (1961), capitalists use the prevailing prices and wages to evaluate new techniques of production. Since wages are likely to change over time, capitalists might factor this into their decision-making process. Basu (2010) introduced an exogenously growing real wage rate to depart from Okishio (1961). Faced with a new technique, capitalists evaluated its expected profitability at the new real wage rate that would result from its exogenous growth (that capitalists know of). In this paper, I explore a slightly different dimension: the possible impact of technical change on the labor market.

Even with an exogenously growing real wage rate, capitalists do not take account of the *possible impact of technical change* on the wage rate when evaluating a newly available technique of production. While this is not an unreasonable assumption given that capitalists act individually and are unlikely to fully understand the impact of technical change at the aggregate level, neither is it unreasonable to think that some capitalists might take account of the impact of technical change at the aggregate level on the labor market when evaluating new techniques of production. For instance, if the new technique of production is strongly labor-saving, capitalists might take into account the fact that adoption of the new technique at the aggregate level is likely to reduce the demand for labor, which, in turn, might reduce upward pressure on real wages.

In this paper, I allow this channel to work by allowing capitalists to form expectations about the future trajectory of the real wage rate when evaluating a new technique of production. The expected real wage captures capitalists' understanding of the possible impact of technical change on the labor market. I do not explicitly model the process of expectation formation, but just consider three mutually exclusive and exhaustive possibilities: real wages are expected to rise, stay unchanged or fall (while remaining positive). Note that this formulation nests the baseline case of Okishio (1961), where wages were expected to not change after technical change, as a special case.

The choice of technique proceeds as follows: When a new technique of production becomes available, capitalists compute an *expected* rate of profit, using the real wage rate they expect to prevail after technical change (which captures their understanding of the impact of aggregate technical change on the labor market). They compare the expected profit rate with the current equilibrium profit rate, which would prevail if the existing technique continued to be used (in which case the real wage rate would also remain unchanged). Only if the expected profit rate is higher than the current current equilibrium profit rate do they adopt it.

I study three closures of the basic model by specifying alternative labor market behavior.[2] The first closure treats the real wage rate as fixed in the long run. This closure is relevant for studying a labor surplus economy, viewed either as a nascent capitalist economy undergoing what Marx termed "primitive accumulation" (Dobb, 1945, page 111) or as a dual-economy studied by classical development economics (Lewis, 1954). One way to characterize this economy is to think of it as being made up of two sectors, a traditional, non-capitalist sector with excess labor and a modern sector driven by profit maximization and capital accumulation. The modern sector is able to attract labor from the traditional sector at a fixed real wage (which is slightly higher than average income in the traditional sector) so long as labor reserves have not been exhausted.

[2]These model closures are also studied in Blecker and Setterfield (2019, chapter 2.4–2.6). My interpretation of the substantive meaning of the alternative closures and my attempt to connect back to Maurice Dobb's pioneering work and to classical development economics is of course different.

The second closure treats the wage share, instead of the real wage rate, as constant in the long run. Dobb characterizes this closure as

> ... a state of the labor market ... in which "relative over-population" is small and in the process of being exhausted by the expansion of industry ... and the competition of capital to obtain labour-power will create a tendency for the price of labour-power to rise ... (Dobb, 1945, page 114).

Calling it the *classical conventional wage share model*, Foley et al. (1999, 2019) have pioneered the study of this case within heterodox macroeconomics. In fact, they have specified the extent to which the real wage rate must rise: it must rise to the full extent of the rise of labor productivity so that the wage share of national income remains unchanged. Therefore, this closure is relevant for studying advanced capitalist economies with strong labor, i.e. where political and institutional factors favorable to labor ensure that the real wage rate grows in tandem with labor productivity. For many decades, right up to the end of the 1970s, the wage share remained relatively stable across advanced capitalist countries. Therefore, the constant wage share model remains relevant for studying this period of capitalist history.

The third, and final, closure allows endogenous adjustment of the real wage rate so as to equalize the growth rates of the supply of and demand of labor, thereby keeping the unemployment rate constant in the long run.[3] This closure is relevant for studying advanced capitalist economies with weakened power of labor, i.e. where political and institutional factors favorable to labor have been eroded. This closure is different from the second one because it does not force the real wage rate to grow at the same rate as labor productivity (to keep the wage share constant), as would be necessary in the second closure. Thus, this closure allows stagnant, falling or slowly growing real wages. A large body of literature has documented the decline in the wage share across numerous countries in the world since

[3] A special case of this closure is the full employment model studied in Kaldor (1961) and in Foley et al. (1999).

the early 1980s (IMF, 2017). Therefore, this closure seems to be particularly relevant for studying contemporary, globalized capitalism.

The *first contribution* of this paper is a methodological one. I present a methodology to study the impact of viable technical change on the equilibrium profit rate in models of economic growth. Using this methodology, a researcher would first use the model equations to solve for the equilibrium rate of profit. In the next step, the researcher would construct two curves, the constant equilibrium profit rate (CEPR) curve and the capitalist viability (CVB) curve.[4]

The CEPR curve gives all configurations of technical change that would keep the equilibrium profit rate unchanged after technical change. Using this, the researcher is able to identify the set of all new techniques of production that would reduce the equilibrium rate of profit. The CVB curve gives all configurations of technical change that would keep the expected profit rate unchanged, given capitalists' expectations about the trajectory of the real wage rate after technical change. Using this, the researcher is able to identify the set of new techniques of production that would be adopted—given the expectations of the capitalists about the future trajectory of the real wage rate.

In the third, and final step, the researcher needs to look at the intersection of the set of viable techniques of production (which depends on capitalists' expectation about the real wage rate) and the set of techniques of production that will reduce the equilibrium rate of profit. If this intersection is nonempty, the researcher can conclude that the LTFRP can hold; if it is empty, she can rule out the LTFRP.

The *second contribution* of this paper is substantive. I implement the above methodology for classical-Marxian growth models with the three alternative closures that I have discussed above. The three closures are meant to capture very different structural and institutional features of capitalist economies: a labor surplus developing economy, an advanced capital-

[4]For models with linear relationships, these curves would be straight lines, as is the case in this paper.

ist economy with strong protections for labor, and an advanced capitalist economy where protections for labor have eroded significantly. Through this analysis, I find the following results.

First, given that technological change in capitalist economies has a pronounced labor bias, i.e. it tends to save on labor and thereby increase labor productivity over time, the possibility of the LTFRP is tied very closely to what Duncan Foley and Tom Michl have termed *Marx-biased technical change* (MBTC) (Foley et al., 1999). This refers to a pattern of technological change marked by an increase in labor productivity and a decline in capital productivity. Through my analysis, I see that this type of technological change is the only serious contender that can support Marx's LTFRP; other types of viable technical change cannot lead to a fall in the equilibrium rate of profit.

Second, whether the LTFRP can hold depends both on the model closure and on capitalists' expectation about the trajectory of the real wage rate after technical change. In the model of a labor surplus developing economy and in the model of an advanced capitalist economy with weak labor, the equilibrium profit rate can fall only if capitalists expected the real wage rate to fall after technical change when evaluating new techniques of production. On the other hand, in the model relevant for an advanced capitalist economy with strong labor, the equilibrium profit rat can fall irrespective of what capitalists believe about the trajectory of the real wage rate when evaluating new techniques of production.

I do not model the process of expectation formation. Hence, I do not take a strong stand on whether capitalists would expect the real wage rate to rise, remain unchanged or fall after technical change. One plausible scenario is that capitalists use the past historical record to form expectation about how the real wage will behave after technical change. If that is the case then capitalists will expect the real wage rate to: (a) remain unchanged in a labor surplus economy; (b) rise in an advanced capitalist economy with strong labor; and (c) not rise in an advanced capitalist economy with weak labor.

Therefore, my analysis shows that the possibility of the LTFRP obtaining is weakest in a labor surplus economy, a little more stronger in an an advanced capitalist economy with weak labor (which would occur if capitalists expect the real wage to fall after technical change), and strongest in an advanced capitalist economy with strong labor. This conclusion seems to be largely consistent with the historical record of the U.S. economy.[5] Moreover, it hearkens back to Maurice Dobb's analysis of Marx's LTFRP, where, in my understanding, the relative strength of labor vis-a-vis capital plays a central role. The alternative model closures of this paper are meant to highlight and capture this latter aspect.

The rest of this paper is organized as follows: in section 2, I present a classical-Marxian model of economic growth; in section 3, I discuss the issue of choice of technique by capitalists and derive the CVB line; in section 4, I investigate configurations of technological change that keeps the equilibrium profit rate constant and derive the CEPR line; in section 5, I discuss the main question of this paper about the movement of the equilibrium profit rate after viable technical change; I conclude the paper in section 6. Proofs are collected in the appendix.

2 A Classical-Marxian growth model with alternative closures

2.1 The basic model

2.1.1 Capital-constrained production

I consider a one-sector closed capitalist economy without government. Let Y, N, L, and K denote real output, the labor force, level of employment and real capital stock, respectively; let

[5]The declining trend in the time series of the U.S. profit rate was reversed in the early 1980s. For some graphical evidence, choose to plot any profit rate series from 1945 to 2023 with the loess trend in this *U.S. Profitability* dashboard `https://dbasu.shinyapps.io/Profitability/`

Y_p, Y_N, and Y_K denote the potential output (or maximum feasible output), full-employment output and full-capacity output, respectively.

Production is characterized by a constant coefficient Leontief technology, defined as follows:

$$Y_p = \min\left(Y_N, Y_K\right), \quad Y_N = \frac{N}{a_0}, \quad Y_K = \frac{K}{a_1}. \tag{1}$$

where a_0 denotes the reciprocal of labor productivity, and a_1 represents the reciprocal of capital productivity. For a given configuration of technology, a_0 and a_1 are strictly positive and constant.

Following the classical-Marxian vision, I assume that the economy is capital constrained, i.e., capital and not labor is the binding constraint. I capture this with the assumption that the maximum feasible output produced with the full capital stock is lower than what could be produced by employing the full labor force,

$$Y_K \leq Y_N. \tag{2}$$

Using the definition of capacity utilization rate as the ratio of actual and potential output,

$$u = \frac{Y}{Y_p},$$

we have $0 < u \leq 1$, and on using (1) and (2), we get

$$Y = uY_p = uY_K = \frac{uK}{a_1}. \tag{3}$$

Since a_0 is constant (and does not depend on the level of output produced), the level of employment is given by

$$L = a_0 Y = \frac{u a_0 K}{a_1}. \tag{4}$$

Thus, technology captured by the two parameters, a_0 and a_1, and the capital stock, K, determine both output and employment.

There are two important trade-offs in this economy, the first between wages and profits, and the second between growth and consumption. The first trade-off is represented by a wage-profit frontier; and the second is represented by a consumption-growth frontier.

2.1.2 Wage-profit frontier

Looked at from the income side, the total value of output is equal to the sum of wage income and profit income,

$$PY = WL + rPK,$$

where P, W, and r denote the price level, the nominal wage rate, and the profit rate, respectively. Dividing through by PY and denoting the real wage rate by $w = W/P$, we get the wage-profit frontier,

$$w = \frac{1}{a_0} - \frac{a_1}{ua_0} r. \tag{5}$$

Since $a_1/(ua_0) > 0$, this gives us a negative relationship between the real wage rate and the profit rate. The wage-profit frontier captures the conflict of interest between the workers (who earn wage income) and the capitalists (who earn profit income) conditional on technology (captured by a_0 and a_1) and demand (captured by u).

2.1.3 Consumption-growth frontier

Looked at from the expenditure side, the total value of output is equal to the sum of consumption and investment expenditure,

$$PY = PC + PI,$$

where C, I denote total real consumption and investment expenditure, respectively. Dividing through by PY and denoting the growth rate of capital stock by $g = \Delta K/K = I/K$, we get the consumption-growth frontier,

$$c = \frac{1}{a_0} - \frac{a_1}{ua_0} g, \tag{6}$$

where c represents real consumption per employed worker. Since $a_1/(ua_0) > 0$, this gives us a negative relationship between consumption and growth. For a given configuration of technology given by a_0 and a_1, and the state of demand by u, the growth rate of capital can be increased only by reducing consumption and increasing accumulation.

2.1.4 Accumulation function

In the classical-Marxian vision, capital accumulation is driven by capitalists, who accumulate (i.e. save and invest) a constant fraction of their profit income in augmenting the capital stock. This is captured by an accumulation function,

$$g = \begin{cases} s_r \left(r - r_m \right), & \text{if } r > r_m \geq 0, \\ 0, & \text{if } r \leq r_m, \end{cases} \tag{7}$$

where g is the rate of growth of the capital stock, r_m denotes the minimum profit rate that is necessary to induce capitalist firms for undertaking positive investment.[6] If the actual profit rate falls to or below r_m, capitalists consume all of their profit income and do not invest. When the profit rate rises above r_m, capitalist invest a fraction, s_r, of their profit income, where $0 < s_r < 1$.[7] The fraction s_r can be called the *accumulation propensity* of capitalists.

[6] I ignore depreciation of the capital stock. Hence, $g = \Delta K/K = I/K = S/K$, where I is investment and S is savings.

[7] When $r > r_m \geq 0$, total profit income is $rPK = (r - r_m)PK + r_m PK$. Capitalists invest $s_r(r - r_m)PK$ and consume the rest, $(1 - s_r)(r - r_m)PK + r_m PK$.

2.1.5 Long run equilibrium

In the classical-Marxian vision, the long run equilibrium of a capitalist economy is defined by the capacity utilization rate, u, converging to the desired or normal rate, u_n. We can capture this with the specification that

$$\widehat{u} = f(u - u_n), \quad f(0) = 0, f'(.) < 0,$$

where $\widehat{u} = \dot{u}/u$. The economy reaches its long run equilibrium when $\widehat{u} = 0$.[8] If, for convenience, the normal rate of capacity utilization is assumed to be unity, then long run equilibrium is given by

$$u = u_n = 1. \tag{8}$$

What I have outlined above is the basic classical-Marxian growth model. It has five endogenous variables: real wage rate, w; consumption per worker, c; growth rate of capital stock, g; profit rate, r; the capacity utilization rate, u; and the following exogenous variables: a_0, a_1, s_r, r_m. The endogenous variables are related through the following four equations explained above: (5), (6), (7), and (8). Thus, we need at least one more equation to close and solve the model. This allows us to use alternative closures of the model.

2.2 Alternative closures

2.2.1 Labor surplus developing economies

The first closure I consider is where the real wage rate remains unchanged,

$$w = \bar{w}, \tag{9}$$

[8] Various mechanisms have been proposed in the literature that can ensure convergence of the capacity utilization rate to the normal rate in the long run. Two prominent mechanisms are changes in the saving propensity through retention of profits by firms (Shaikh, 2009), and inflation stabilization by a Central Bank (Duménil and Lévy, 1999; Michl, 2024).

and where $\bar{w}$ is a constant. This closure is relevant for a labor surplus economy, like one studied extensively in classical development economics (Lewis, 1954); it can also be understood as a developing economy undergoing capitalist development through what Marx called primitive accumulation (Dobb, 1945). This assumption was used in the 'first' classical model of economic growth with unlimited supply of labor in Kaldor (1961).

Drawing on insights from classical development economics, we can characterize a labor surplus economy as being composed of two sectors, a modern, capitalist sector and a traditional, non-capitalist sector. The latter is populated by a very large number of small-scale peasant producers working on their farms with family labor and generating a low level of income. As long as $\bar{w}$ is above the average peasant income, the capitalist sector can draw out the surplus labor from the traditional sector with very little pressure on wages and without reducing agricultural output. This feature is captured by (9).

To summarize, the classical-Marxian growth model for a labor surplus developing economy has five endogenous variables, w, c, g, r, and u, related to each other by five equations, (5), (6), (7), (8) and (9). To ensure meaningful equilibrium solutions of this model, we need the following restrictions.

Assumption 1 *The parameters $a_0, a_1, r_m, \bar{w}$ are strictly positive, $0 < s_r < 1$, and the feasible region of technological possibilities is given by*

$$a_0 < \frac{1}{\bar{w}}, \quad a_1 < \frac{1}{r_m} - \left(\frac{\bar{w}}{r_m}\right) a_0. \tag{10}$$

The first part of the assumption states that labor productivity, $1/a_0$, must be larger than the fixed real wage rate, $\bar{w}$; if labor productivity fell below this level, then capitalist production would not be feasible because output would be lower than the wage rate (so that profit income would be negative). The second part gives a lower bound for the capital productivity, $1/a_1$, that keeps the profit rate above the minimum threshold, r_m. If capital productivity fell below this level, given a_0, then the profit rate would fall below the threshold and the economy would grind to a halt.

2.2.2 Advanced capitalist countries with strong labor

The second closure I consider is where the wage share (ratio of wage income to total income) or equivalently the profit share (ratio of profit income to total income) remains constant,

$$\pi = \bar{\pi}, \tag{11}$$

and where $0 < \bar{\pi} < 1$ is a constant. This model has been called the *conventional wage share model* by Foley et al. (1999). It is applicable to an advanced capitalist economy which is no longer characterized by surplus labor and, in addition, where labor is powerful enough to force the real wage rate to grow in tandem with labor productivity.

In advanced capitalist countries, the wage share has remained constant for long periods of time (running right up to the end of the 1970s). In the early 1960s, this led Nicholas Kaldor to present this as one of the 'stylized facts' of economic growth in capitalism (Kaldor, 1961). The classical conventional wage share model incorporates this stylized fact. Therefore, it remains an important version of the classical-Marxian growth model to study advanced capitalist countries.

To summarize, the classical-Marxian growth model for an advanced capitalist country with strong labor has five endogenous variables, w, c, g, r, and u, related to each other by five equations, (5), (6), (7), (8) and (11). I need to impose the following restrictions to ensure meaningful equilibrium solutions of this model.

Assumption 2 *The parameters a_0, a_1, r_m are strictly positive, $0 < \bar{\pi} < 1$, $0 < s_r < 1$ and* *the feasible region of technological possibilities is given by*

$$a_1 < \frac{\bar{\pi}}{r_m}. \tag{12}$$

This assumption gives a lower bound for the capital productivity, $1/a_1$, that keeps the profit rate above the minimum threshold, r_m. If capital productivity fell below this level, given a_0, then the profit rate would fall below the threshold and the economy would stop growing. This assumption rules that out.

2.2.3 Advanced capitalist countries with weak labor

There is a growing body of evidence which shows that the pattern of constant wage share, which had been observed for many decades, changed in the early 1980s (IMF, 2017). Since then the wage share has declined, in both advanced capitalist and developing capitalist economies. Political and institutional factors that had forced real wages to grow at the same rate as labor productivity seems to have been weakened by technological change, globalization of production, decline in unionization rates and other such factors. Hence, it seems theoretically desirable to allow for closures of the classical-Marxian growth model where the wage share is not held constant *as an assumption*. An alternative stylized fact might recommend itself: over the long run, the unemployment rate is constant. We can capture this assumption and generate another closure of the model by equating the growth rates of the supply of and demand for labor.

Labor supply: Following Marx, I assume that in capitalist economies, labor supply is socially determined. At the very least, labor supply can be augmented by labor-saving technical change and by drawing latent reserves of labor (within the household or in small-scale agriculture) into the orbit of profit-oriented commodity production. Both processes of labor supply growth are impacted by the real wage rate. On the one hand, real wage pressures induce capitalists to search for and adopt labor-saving technical change; on the other hand, high real wage rates increase the incentive to supply household and petty-commodity production labor to the capitalist labor market. Hence, a non-negative response of labor supply growth to the real wage rate seems a natural way to model these features of capitalist labor markets,

$$n = n_0 + n_1 w, \tag{13}$$

where n is the growth rate of labor supply, $n_0 > 0$ represents the base growth rate (driven by population growth rate, immigration, etc.), and $n_1 \geq 0$ captures the response of labor supply growth to real wage increases.[9] Different values of n_1 can be used to capture different sub-cases, which I will comment on below.

Labor demand: Recall that for a given configuration of technology, a_0 (reciprocal of labor productivity) and a_1 (reciprocal of capital productivity) are constant. Hence, the growth rate of capital stock is equal to the growth rate of output, which in turn, is equal to the growth rate of the demand for labor. These three growth rates are given by g. On the other hand, the growth rate of the supply of labor is given by n in (13), which responds to the level of the real wage rate, w.

Adjustment and equilibrium: The real wage rate is the adjustment variable, which changes to establish equilibrium in the labor market in the long run. The growth rate of the real wage rate responds positively to the difference between the growth rates of the demand for and supply of labor,

$$\widehat{w} = \phi\left(g - n\right), \qquad \phi\left(0\right) = 0, \phi'\left(.\right) > 0,$$

and a steady state, long run equilibrium of the model is obtained when $\widehat{w} = 0$, i.e.

$$g = n. \tag{14}$$

[9]To keep the analysis simple, I do not distinguish between the total population and the labor force.

It is important to note that the equality of the growth rates of labor demand and supply does not imply full employment in equilibrium; it only implies that the unemployment rate will remain constant in equilibrium. For instance, if there was unemployment in the initial period, then the equilibrium unemployment rate will be the same as that rate.

To summarize, the classical-Marxian growth model for advanced capitalist economies with weak labor has six endogenous variables, w, c, g, r, u, and n, which are linked by six equations, (5), (6), (7), (8), (13) and (14). To ensure meaningful equilibrium solutions of this model, I need the following restrictions.

Assumption 3 *The parameters a_0, a_1, n_0 are all strictly positive, the parameters n_1, r_m are weakly positive, $0 < s_r < 1$, and the reciprocal of capital productivity is bounded above, i.e.*

$$a_1 < a_1^{max} = \frac{s_r}{n_0 + s_r r_m}. \tag{15}$$

This assumption is similar to Assumption 2. It gives a lower bound for the capital productivity, $1/a_1$, that keeps the profit rate above the minimum threshold, r_m, so that we can rule out zero growth.

2.3 Equilibrium profit rate

For each of the models described above, we can solve for the equilibrium profit rate, r^*. For the labor surplus developing economy, we have

$$r^* = \frac{1 - a_0 \bar{w}}{a_1}. \tag{16}$$

Thus, the equilibrium profit rate is impacted by the technological parameters, a_0 and a_1, and the exogenous real wage rate, $\bar{w}$.

For the advanced capitalist country with strong labor, we have

$$r^* = \frac{\bar{\pi}}{a_1}, \tag{17}$$

so that the equilibrium profit rate is impacted by the single technological parameter a_1 and the exogenous profit share, $\bar{\pi}$.

For the advanced capitalist economy with weak labor, we have

$$r^* = \frac{a_0\left(n_0 + s_r r_m\right) + n_1}{a_0 s_r + a_1 n_1}, \tag{18}$$

so that the equilibrium profit rate is impacted by the accumulation propensity, s_r, the minimum threshold profit rate for positive investment, r_m, the parameters of the labor supply growth function, n_0 and n_1, and most crucially for the analysis of this paper, on the technology parameters, a_0 (reciprocal of labor productivity) and a_1 (reciprocal of capital productivity).

The fact that, in each case, the equilibrium profit rate is impacted by the technology parameters a_0 and a_1 is very important. It implies that technological change *can* have an impact on the equilibrium profit rate. Therefore, it opens up the possibility that Marx's LTFRP *can* hold. Conversely, all special cases of the models which make the equilibrium profit rate independent of the technology parameters will rule out the possibility of Marx's LTFRP.

2.4 Equilibrium values of other endogenous variables

The equilibrium values of the other four endogenous variables are not directly relevant for the analysis presented in this paper. But they can be easily calculated using the equations of the relevant model and the expression for the relevant equilibrium profit rate:

$$g^* = s_r \left(r^* - r_m \right), \tag{19}$$

$$w^* = \frac{1}{a_0} - \frac{a_1}{a_0} r^*, \tag{20}$$

$$c^* = \frac{1}{a_0} - \frac{a_1}{a_0} g^*. \tag{21}$$

My main interest in using the classical-Marxian model of economic growth is in studying the effect of technical change on the equilibrium profit rate.[10] I will do so in two steps. In the first step, I will investigate the viability condition, i.e. the condition under which capitalist firms will adopt a new technique of production that becomes available; in the second step, I will study how the actual equilibrium profit rate changes if some new technique of production is adopted. Bringing these two together will then allow me to answer the main question of this paper: under what conditions can the LTFRP hold? But before I turn to that, I want to ensure that the model I am working with produces meaningful equilibrium solutions.

2.5 Meaningful equilibrium solutions

While describing various closures of the classical-Marxian model, I have imposed restrictions on parameters to ensure that meaning solutions are guaranteed.[11] By meaningful equilibrium solutions, I mean the following: the equilibrium profit rate is positive and larger than the

[10]For illuminating diagrammatic analyses of the effect of changes in exogenous parameters on all endogenous variables, see Blecker and Setterfield (2019, chapter 2.5, 2.6).

[11]Kaldor (1961) also imposes inequality restrictions to generate meaningful equilibrium solutions for some of his models.

minimum required by capitalists to undertake positive investment,

$$r^* > r_m \geq 0, \tag{22}$$

and the equilibrium real wage rate is positive,

$$w^* > 0. \tag{23}$$

The former, in turn, ensures that the equilibrium growth rate is positive, $g^* > 0$, because

$$g^* = s_r(r^* - r_m);$$

and the latter implies that consumption per worker is positive, $c^* > 0$, because, using (5) and (6), we see that

$$c^* - w^* = \frac{a_1}{a_0}\left(r^* - g^*\right),$$

which, on using $g^* = s_r(r^* - r_m)$, becomes,

$$c^* - w^* = \frac{a_1}{a_0}\left((1 - s_r)r^* + s_r r_m\right) > 0.$$

Proposition 1 *If Assumption 1 holds, then the model of the labor surplus developing economy produces meaningful equilibrium solutions.*

Proposition 2 *If Assumption 2 holds, then the model of the advanced capitalist economy with strong labor produces meaningful equilibrium solutions.*

Proposition 3 *If Assumption 3 holds, then the model of the advanced capitalist economy with weak labor produces meaningful equilibrium solutions.*

3 Choice of technique

Suppose the economy starts in the long run equilibrium position with a technique of production given by (a_0, a_1). In this equilibrium position, the profit rate is given by r^* in (16), (17) or (18), depending on which model is relevant. Now, suppose a new technique of production, given by (a'_0, a'_1), becomes available; suppose further that this new technique of production satisfies Assumption 1, 2 or 3, whichever is relevant. Will a capitalist adopt this new technique of production?

3.1 Importance of capitalists' expectation

The existing literature on the choice of technique has largely followed the pioneering contribution of Okishio (1961) in assuming that capitalists evaluate the new technique at the existing real wage rate. This seems unnecessarily restrictive because real wage rates change over time and capitalists would know this fact. One way to go beyond Okishio (1961) is to allow the real wage rate to grow at an exogenous rate, as introduced in Basu (2010). While this is a step in the direction of generalizing Okishio's analysis, it misses out on a crucial dimension: the *possible impact of technical change itself on the real wage rate*.

While it is no doubt true that capitalists make individual decisions and therefore are unable to *fully* take account of aggregate level changes, e.g. technical change, it is not unreasonable to assume that they might form expectations of those effects and allow such expectations to have an impact on their choices. Consider the case of a new technique of production that saves on the labor input by a large amount. If such a technique were to be adopted at the aggregate level, a capitalist might reason, it is likely to reduce the demand for labor and thereby prevent wage pressures from building up in the economy after technical change. The individual capitalist might then take this possible effect of aggregate technical change on the labor market into account while evaluating the new technique of production.

I will capture the effect of capitalist's expectations about the future trajectory of the real wage rate, which is based on her understanding of the possible impact of technical change on the labor market, on her choice of technique with the following assumption.

Assumption 4 *Capitalist firms believe that, if the new technique of production is adopted, then the new equilibrium wage rate will become* βw^* *where* $0 < \beta < \infty$*, and* w^* *is the current equilibrium real wage rate.*

Here I have not specified how exactly capitalists form expectations about the impact of technical change on the real wage rate. No matter how they form those expectations, we need to consider only three mutually exclusive and exhaustive possibilities: $\beta > 1, \beta = 1$, and $\beta < 1$. These possibilities refer to situations where capitalists expect the real wage rate to increase, remain unchanged and decline (while remaining positive), respectively, *if the new technique of production is adopted at the aggregate level.*

It is important to note that Assumption 4 is not restrictive. It nests the baseline case where capitalists do not expect any change in the real wage rate after technical change with the assumption $\beta = 1$. But it is more general because it allows other possibilities, either when capitalists expect the real wage to decline or expect it to increase.

3.2 The capitalist viability condition

Given a value of β (reflecting the expectations of capitalist firms), we can use (5) to find the profit rate that a capitalist firm can *expect to* earn when the new technique of production is adopted as

$$r^e(\beta) = \frac{1 - a_0' \beta w^*}{a_1'},$$

where r^e, which denotes the expected profit rate, is a function of β. If the capitalist firm does not adopt the new technique, and all other firms do the same, then the existing technique continues to be used and there will be no effect on the real wage rate. The current equilibrium profit rate will continue to prevail.[12] Hence, the capitalist firm, while evaluating the new technique of production, will compare the expected profit rate with the current equilibrium profit rate. It will adopt the new technique only if the expected profit rate exceeds the current equilibrium profit rate. Thus, the capitalist firm will adopt the new technique if $r^e(\beta) > r^*$, i.e. if the following capitalist viability condition is satisfied,

$$r^e(\beta) = \frac{1 - a_0'\beta w^*}{a_1'} > r^*,$$

which, since $w^* = (1 - a_1 r^*)/a_0$, can be rearranged to give

$$a_1' < \frac{1}{r^*} - \left(\frac{\beta - \beta a_1 r^*}{a_0 r^*}\right) a_0'. \tag{24}$$

Using the expression for the equilibrium profit rate, r^* in (16), (17) or (18), depending on which model is relevant, we get the viability condition in each model. Thus, the viability condition for the model of the labor surplus developing economy becomes

$$a_1' < \frac{a_1}{1 - a_0\bar{w}} - \left(\frac{a_1\beta\bar{w}}{1 - a_0\bar{w}}\right) a_0'. \tag{25}$$

The corresponding viability condition in the model for an advanced capitalist economy with strong labor is given by

$$a_1' < \frac{a_1}{\bar{\pi}} - \left[\frac{a_1\beta(1 - \bar{\pi})}{a_0\bar{\pi}}\right] a_0'. \tag{26}$$

and the viability condition in the model for an advanced capitalist economy with weak labor

[12] In effect, I am treating the individual capitalist as a representative agent, a representative of the whole capitalist class. The divergence between individual and collective (class) interests is an important part of capitalist society. This aspect should be explored in future research.

is given by

$$a_1' < \left(\frac{a_0 s_r + a_1 n_1}{a_0 n_0 + a_0 s_r r_m + n_1}\right) + \left[\frac{\beta\left(a_1 n_0 + a_1 s_r r_m - s_r\right)}{a_0 n_0 + a_0 s_r r_m + n_1}\right] a_0'. \tag{27}$$

For future reference let me note that I will call a straight line on the (a_0', a_1') plane given by (25), (26) and (27), respectively, with an equality instead of the inequality, as the *capitalist viability condition* (CVB) line in those models. All configurations of new techniques of production that lie below the CVB line are viable.

4 Constant equilibrium profit rate

If technical change represented by the new technique of production, (a_0', a_1'), is adopted, let the equilibrium profit rate be denoted r^{**}. It can be computed by replacing (a_0, a_1) with (a_0', a_1') in (16), (17) or (18), depending on which model is relevant.

I will call the locus of configurations of technical change that keeps the equilibrium profit rate unchanged the *constant equilibrium profit rate* (CEPR) line. It is given by the combinations of (a_0', a_1') such that $r^{**} = r^*$.

For the model of the labor surplus developing economy, the CEPR line is given by

$$a_1' = \frac{a_1}{1 - a_0 \bar{w}} - \left(\frac{a_1 \bar{w}}{1 - a_0 \bar{w}}\right) a_0'. \tag{28}$$

For the model of the advanced capitalist economy with strong labor, the CEPR line is given by

$$a_1' = a_1; \tag{29}$$

finally, the CEPR line for the model of the advanced capitalist economy with weak labor is given by

$$a_1' = \left(\frac{a_0 s_r + a_1 n_1}{a_0 n_0 + a_0 s_r r_m + n_1}\right) + \left[\frac{(a_1 n_0 + a_1 s_r r_m - s_r)}{a_0 n_0 + a_0 s_r r_m + n_1}\right] a_0'. \tag{30}$$

From the expression for the equilibrium profit rate in (16), (17) and (18), it is immediately clear that if a_1' increases while holding a_0' fixed, then the equilibrium profit rate after technical change falls. Thus, all points above the CEPR line represent situations where the equilibrium profit rate declines after technical change; all points below this line represent scenarios where the equilibrium profit rate rises after technical change. Points on the CEPR line, of course, keep the equilibrium profit rate unchanged.

5 Technical change and profitability

I am now ready to answer the main question of this paper: when will the equilibrium rate of profit fall after viable technical change? For each model, I will answer this question in two steps. First, I will draw the CVB and CEPR lines on the same graph. Second, I will identify the intersection of two sets: (a) the set of viable techniques of production (all points below the CVB line) and (b) the set of techniques of production that reduce the equilibrium rate of profit (all points above the CEPR line). If the intersection is non-empty, I will conclude that the LTFRP can hold. If the intersection is empty, I will conclude that the LTFRP cannot hold.

5.1 Labor surplus economy

The model of the labor surplus developing economy can be analyzed with Figure 1. Note, first of all, that Assumption 1 holds and hence the economy must always lie in the interior of the triangle OFG. The point E denotes the economy before technical change, i.e. with technique of production (a_0, a_1).

The two crucial lines in Figure 1 are the CEPR and CVB lines. So, let us compare the equation for the CEPR line in (28) and the equation for the CVB line in (25). We can note two things immediately: (a) the vertical intercept is the same for the two lines; (b) the slope of the CVB line is β times the slope of the CEPR line.

Let us now turn to the CEPR lines in Figure 1, AC. If the economy is on the line AC, then the equilibrium profit rate does not change after technical change. If the economy ends up above (below) the line AC, then the profit rate declines (increases) after technical change.[13]

Turning to the CVB line given by (25), we see three different scenarios represented in Figure 1. If capitalists expect the equilibrium wage rate to rise after technical change, i.e. $\beta > 1$, and use this expectation to inform their decision about adopting new techniques of production, then the CVB is represented by the line AB. All points *inside* the triangle AOB represent viable techniques of production. In a similar manner, we get the CVB lines as AC and AD when $\beta = 1$ and $\beta < 1$, respectively.

Now that we have put the CEPR and CVB lines on the same graph in Figure 1, let us address the key question: when can viable technical change lead to a fall in the equilibrium rate of profit? The answer rests crucially on the type of technical change *and* on capitalist expectations about the trajectory of the real wage rate.

Let us start with the first aspect. The vertical and horizontal dashed lines passing through the point E demarcates four types of technical change: labor-using, capital-using (Northeast); labor-using, capital-saving (Southeast); labor-saving, capital-using (Northwest); labor-saving, capital-saving (Southwest). The historical record of capitalism clearly rules out labor-using technical change in the long run. Hence, the real alternatives to consider are labor-saving, capital-using (Northwest) and labor-saving, capital-saving (Southwest) techni-

[13] The point E represents the original technique of production, (a_0, a_1). It is easy to verify from (28) that this point lies on the CEPR line.

cal changes.

In Figure 1, we see an important fact: labor-saving, capital-saving (Southwest) technical change cannot lead to a fall in the equilibrium rate of profit. This is because all possible configurations of this pattern of technical change lies wholly below the CEPR line. Hence, the equilibrium rate of profit can only rise if such a technical change were to be adopted.[14] Therefore, I reach an important conclusion: the only possible candidate viable technical change that can lead to a LTFRP is of the labor-saving, capital-using type. This is precisely what Foley et al. (1999) have termed MBTC.

Now we come to the second aspect: capitalists' expectation. If capitalists expect the equilibrium wage rate to remain unchanged, in which case $\beta = 1$, then the CVB is represented by the line AC (so that the CEPR and CVB lines coincide). The menu of viable techniques of production is now represented by the interior of the triangle AOC. In this case, the intersection of the set of viable techniques of production and the set of techniques of production that reduce the equilibrium rate of profit is empty. Thus, the LTFRP cannot hold even if we limit ourselves to MBTC.

If capitalists expect the equilibrium wage rate to increase, in which case $\beta > 1$, then the CVB is represented by the line AB. The set of viable techniques of production is now represented by the interior of the triangle AOB. In this case, once again, the intersection of the set of viable techniques of production and the set of techniques of production that reduce the equilibrium rate of profit is empty. Thus, once again, the LTFRP cannot hold even if we limit ourselves to MBTC.

If capitalists expect the equilibrium wage rate to fall after technical change, i.e. they use $\beta < 1$ while considering the new technique of production, then the CVB is represented by the line AD. Viable techniques of production are now represented by the interior of the triangle

[14] It should be noted that not all labor-saving, capital-saving are always viable. For instance, if $\beta > 1$, only a fraction of all labor-saving, capital-saving technical changes are viable (which is given by the part of the triangle AOB which lies below the horizontal line through E).

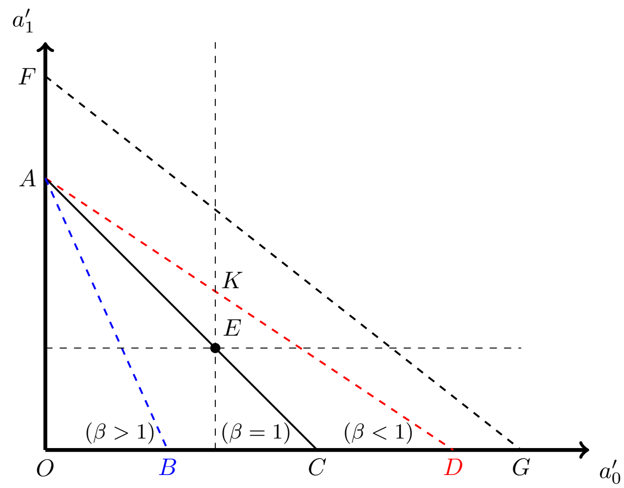

Figure 1: Technical change and profitability in a labor surplus economy. The line AC represents the CEPR curve given by (28). The lines AD, AC and AB represent the CVB line given by (25), for $\beta < 1, \beta = 1$ and $\beta > 1$, respectively. The technologically feasible region, which is given by Assumption 1, is the interior of the triangle OFG, where $OF = 1/r_m$ and $OG = 1/\bar{w}$. E represents the initial configuration of technology (before technical change).

AOD. In this case, the intersection of the set of viable techniques of production and the set of techniques of production that reduce the equilibrium rate of profit is nonempty and is given by the triangle ACD. Thus, it is possible for the LTFRP to hold if technical change is of the Marx-biased type: in this case any new technique of production in the triangle AEK will be adopted and also lead to a decline in the equilibrium rate of profit).

Why does the LTFRP become possible when $\beta < 1$? What is the intuition for this result? Consider the triangle AEK. These represent a subset of MBTC. If adopted, they will reduce the equilibrium rate of profit (because they are above the CEPR line, AC). This is because the actual increase in the non-labor cost outweighs the decline in labor cost for such techniques of production. But if $\beta < 1$, these techniques are viable because the *expected* reduction in labor cost (assuming $\beta < 1$) outweighs the increase in nonlabor costs. Thus, if a new technique of production becomes available in the triangle AEK, then capitalists will adopt it if $\beta < 1$. The new equilibrium profit rate will fall (because the point would be above the CEPR line). Hence, we will have the LTFRP.

Since the results of the analysis depend crucially on β, i.e. whether capitalists expect the real wage rate to rise, remain unchanged or fall after technical change, it needs to be ascertained which of these possibilities is most likely. I do not want to take a strong stand on this issue because expectation formation is a complicated process and I have not modeled it explicitly. One possibility might be plausible: capitalists look to the past to form expectations. If that is the case, then capitalists will expect the real wage rate to remain unchanged after technical change because we are in a labor surplus economy. This would make the $\beta = 1$ case most likely. Hence, the LTFRP is unlikely to hold.

5.2 Advanced capitalist economy with strong labor

The model of the advanced capitalist economy with strong labor can be analyzed with Figure 2. In this case, the CEPR curve given by (29) is represented by the horizontal line at H. The lines AD, AC and AB represent the CVB line given by (26), for $\beta < 1, \beta = 1$ and $\beta > 1$, respectively. Note, once again, that Assumption 2 holds and hence the economy must always lie below the horizontal line at G (only such techniques of production are feasible). The point E denotes the economy before technical change, i.e. with technique of production (a_0, a_1).

The answer to the question as to when the LTFRP can hold changes dramatically. This is because the CEPR line is now given by the horizontal line at H. Thus, all points above the horizontal line at H lead to a fall in the equilibrium rate of profit. Since the CVB lines remain the same as before, we see from Figure 2 that the intersection of the set of viable techniques of production and the set of techniques of production that reduce the equilibrium rate of profit is always nonempty. Thus, as long as we limit ourselves to MBTC, the LTFRP can hold irrespective of whether capitalists expect the real wage rate to rise, stay unchanged or fall after technical change.

What is the intuition for this result? Since the CEPR curve is a flat line at H, any new

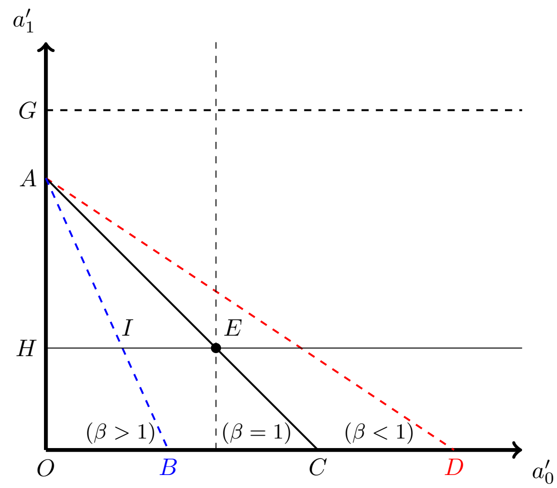

Figure 2: Technical change and profitability in an advanced capitalist economy with strong labor. The horizontal line at H represents the CEPR curve given by (29). The lines AD, AC and AB represent the CVB line given by (26), for $\beta < 1, \beta = 1$ and $\beta > 1$, respectively. The technologically feasible region, which is given by Assumption 2, is demarcated by the points below the horizontal line at G, where $OG = \bar{\pi}/r_m$. E represents the initial configuration of technology (before technical change).

technique which increases a_1 will lead to a fall in the equilibrium rate of profit. This comes from the fact that the real wage has to rise sufficiently to keep the wage share constant, and thus the increase in non-labor cost is not compensated for by an adequate fall in the labor cost. This makes all MBTC open to the LTFRP, irrespective of what β is, i.e. no matter what capitalists believe about the trajectory of future real wage rates. In this closure of the model, the results of the analysis regarding LTFRP does not depend on β. Hence, we do not need to think about which is most probable: $\beta < 1, \beta = 1$ or $\beta > 1$.

5.3 Advanced capitalist economy with weak labor

The model of the advanced capitalist economy with weak labor has two interesting sub-cases depending on whether $n_1 = 0$ or $n_1 > 0$.

5.3.1 Exogenous labor supply

The first sub-case, with $n_1 = 0$, seems to be relevant as a description of a rich capitalist economy, where the wage rate has crossed the threshold beyond which labor supply stops responding to the real wage rate.[15] Using $n_1 = 0$ in (18), we get

$$r^* = \frac{n_0 + s_r r_m}{s_r}.$$

Thus, the profit rate is independent of the technology parameters, a_0 and a_1. Hence, technical change cannot impact the equilibrium rate of profit and hence, there is no possibility of Marx's LTFRP from obtaining. Therefore, I do not analyze this case with the CEPR and CVB lines.

A special sub-case would be when, not only is the growth rate of labor unresponsive to the level of the real wage rate but population growth has itself declined to zero, $n_0 = n_1 = 0$. This might be relevant for a very high income capitalist economy that has achieved zero population growth. In this case, the economy reaches an equilibrium with $r^* = r_m$. Using the accumulation function in (7), we see that the economy's growth rate falls to zero. In effect, capitalists would just invest to replace the depreciated capital stock and the economy would be stationary, i.e. $g = 0$, at a high level of the real wage rate. This is a happy version of Ricardo's stationary state.

5.3.2 Endogenous labor supply

The second case, with $n_1 > 0$, can be relevant for studying an advanced capitalist economy where the real wage rate is still below the threshold beyond which labor supply no longer responds to the real wage rate. This might be relevant to middle-income capitalist economies where labor supply still responds positively to growth in real wages.

[15] Immigration induced increases in the labor supply could be captured by n_0.

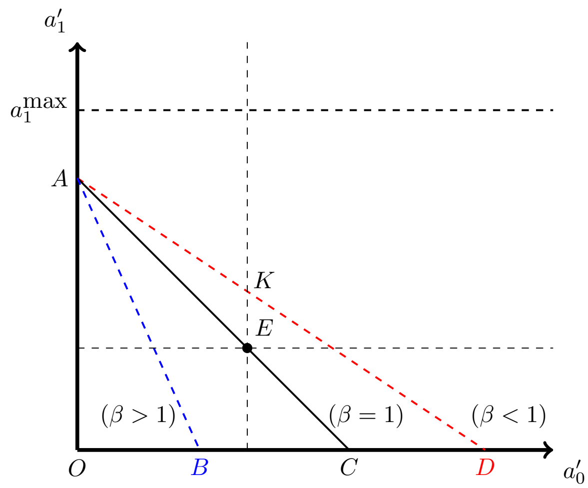

Figure 3: Technical change and profitability in an advanced capitalist economy with weak labor. The line AC represents the CEPR curve given by (30). The lines AD, AC and AB represent the CVB line given by (27), for $\beta < 1, \beta = 1$ and $\beta > 1$, respectively. The technologically feasible region, which is given by Assumption 3, lies below the horizontal line at $a_1^{\max} = s_r / (n_0 + s_r r_m)$.

In this case, the equilibrium profit rate is given by (18) and we can use Figure 3 for the analysis. In Figure 3, the line AC represents the CEPR curve given by (30). The lines AD, AC and AB represent the CVB line given by (27), for $\beta < 1, \beta = 1$ and $\beta > 1$, respectively. Assumption 3 implies that $a_1 n_0 + s_r a_1 r_m - s_r < 0$. Hence, using (30) and (27), we can see that the CEPR line and the CVB line are both downward-sloping, as depicted in Figure 3.

Note, once again, that Assumption 3 holds and hence the economy must always lie below the horizontal line at $a_1^{\max}$. The point E denotes the economy before technical change, i.e. with technique of production (a_0, a_1).

Using Figure 3, we can see that the answers to the question as to when the LTFRP can hold is now similar to the answers given for the model of the labor surplus developing economy. This is because the CEPR line coincides with the CVB line for $\beta = 1$. Hence, if capitalists expect the real wage rate to either increase or remain unchanged after technical change, i.e. $\beta \geq 1$, then there is no possibility of the LTFRP holding, even if we limit ourselves to MBTC; if, on the other hand, capitalists expect the real wage rate to fall after technical change, i.e. $\beta < 1$, then the LTFRP can hold if technical change is of the Marx-biased type. The intuition for this result is exactly the same as for the labor surplus economy.

Since the results of the analysis about the LTFRP depends on β, we are, once again, confronted with the question as to which is most probable: $\beta < 1, \beta = 1$ or $\beta > 1$. Unlike the second closure (mature economy with strong labor), the historical record in this closure would not show robust growth of the real wage rate. Being without adequate social and political protection, labor is not able to force the real wage to rise with labor productivity. This opens up and interesting possibility.

Confronted with a MBTC and forming expectations on the basis of the historical record, a capitalist might legitimately think that the real wage rate will fall after technical change is adopted at the aggregate level, i.e. she can legitimately believe that $\beta < 1$. This is because adoption of MBTC is likely to reduce the demand for labor and therefore reduce wage pressures, she would think. If the political and social protections are inadequate to protect labor, the capitalist might believe that a strong dose of labor-saving technical change (which is part of the MBTC) will, in fact, reduce the real wage. This means that it is possible for Marx's LTFRP to hold—if the new technique of production falls within the triangle AEK.

6 Conclusion

Technical change in capitalist economies consist of at least two stages. In the first stage, new techniques of production are generated by R&D efforts of public and private institutions. In the second stage, capitalist firms evaluate whether a new technique will increase their rate of profit given their expectations about the trajectory of real wages. They adopt the new technique only if it is expected to increase their rate of profit. Such techniques of production can be called capitalistically viable.

Neither are all capitalistically viable techniques socially beneficial, nor are all socially desirable techniques capitalistically viable (Foley, 1986; Basu, 2021). But leaving this issue aside, we can ask whether viable technical change might ever lead to a fall in the equilibrium rate of profit. If this were to happen then it would paradoxically undermine capitalists: even when they only adopt techniques of production that are expected to raise the rate of profit, they end up in the new long run equilibrium with a lower profit rate!

In this paper, I have investigated this question in classical-Marxian models of economic growth with alternative closures. These closures map onto three distinct structural and institutional settings of capitalist economies: a labor surplus developing economy, an advanced capitalist economy where labor has relatively strong bargaining power vis-a-vis capital, and finally, an advanced capitalist economy with weak labor.

The results of my analysis show that whether the equilibrium profit rate falls after viable technical change depends on both the setting of a particular capitalist economy (captured by alternative closure of the model) and on how capitalists expect real wages to behave after technical change. In particular, I show that in both labor surplus economies and in advanced capitalist economies with weak labor, the equilibrium profit can fall only if capitalists base their choice of technique decisions on an expected fall in the real wage rate after technical change. On the other hand, in the model of an advanced capitalist economy with strong labor, I show that the equilibrium profit can always fall, i.e. irrespective of what capitalists expect about the trajectory of the real wage rate when making their choice of technique decision.

The specific way in which I have captured strong labor in an advanced capitalist economy is by keeping the wage share of national income fixed—this is the classical conventional wage share model of Foley et al. (1999). Thus, labor is strong in the specific sense that the real wage rate grows in tandem with labor productivity, and this is the precise context in which the LTFRP becomes possible for MBTC.

While the reasons for the emergence and recurrence of MBTC have been studied within the induced innovation literature (Foley et al., 2019, chapter 7) and within game-theoretic modeling frameworks (Baldani and Michl, 2000; Kang and Rieu, 2009), the question about the constancy of the wage share has attracted much less attention. What plausible political and economic mechanisms might ensure that the wage share remains stable over time? How best to model such processes? These would seem like fruitful questions to pursue within what Tom Michl has aptly called the 'falling rate of profit research program' (Michl, 2023).

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A Appendix: Proofs

A.1 Proof of Proposition 1

Since Assumption 1 holds, we have

$$\frac{1 - a_0 \bar{w}}{r_m} > a_1$$

which is equivalent to

$$\frac{1 - a_0 \bar{w}}{a_1} > r_m$$

because $a_1 > 0$ and $r_m > 0$. Using the definition of the equilibrium profit rate, r^* in (16), this is equivalent to

$$r^* > r_m \geq 0.$$

Moreover, wee know that $w^* = \bar{w} > 0$ by assumption.

A.2 Proof of Proposition 2

Using Assumption 2 and the expression for the equilibrium profit rate in (17), we have

$$r^* = \frac{\bar{\pi}}{a_1} > r_m,$$

because $a_1 > 0$ and $r_m > 0$. Moreover, we have

$$w^* = \frac{1 - \bar{\pi}}{a_0} > 0$$

because $0 < \bar{\pi} < 1$ and $a_0 > 0$.

A.3 Proof of Proposition 3

A.3.1 Two lemmas

I will need two preliminary results.

Lemma 1 *$r^* > r_m$ is equivalent to $a_1 n_1 r_m < n_1 + a_0 n_0$.*

Proof. Using the expression for the equilibrium rate of profit in (18), $r^* > r_m$ is equivalent to

$$\frac{a_0 (n_0 + s_r r_m) + n_1}{a_0 s_r + a_1 n_1} > r_m,$$

which is equivalent to

$$a_0 n_0 + a_0 s_r r_m + n_1 > a_0 s_r r_m + a_1 n_1 r_m,$$

because $a_0 s_r + a_1 n_1 > 0$. Canceling terms on both sides, the above is equivalent to

$$a_1 n_1 r_m < n_1 + a_0 n_0.$$

Lemma 2 *$w^* > 0$ is equivalent to $r_m < (1/a_1) - (n_0/s_r)$.*

Proof. Since $w^* = (1 - a_r r^*)/a_0$,

$$w^* > 0 \quad \Leftrightarrow \quad r^* < 1/a_1.$$

Using the expression for the equilibrium rate of profit in (18), this is equivalent to

$$\frac{a_0\left(n_0 + s_r r_m\right) + n_1}{a_0 s_r + a_1 n_1} < \frac{1}{a_1},$$

which is equivalent to

$$a_1 a_0 n_0 + a_1 a_0 s_r r_m + a_1 n_1 < a_0 s_r + a_1 n_1$$

because $a_0 s_r + a_1 n_1 > 0$. Canceling terms on both sides, dividing through by $a_0 a_1 s_r$ and rearranging, the above is equivalent to

$$r_m < \frac{1}{a_1} - \frac{n_0}{s_r}.$$

because $a_0 > 0, a_1 > 0, s_r > 0$.

A.3.2 Proof of Proposition 3

Since Assumption 3 holds, we have

$$a_1 < \frac{s_r}{n_0 + s_r r_m}.$$

Since $n_0 + s_r r_m > 0, s_r > 0, a_1 > 0$, this is equivalent to

$$\frac{n_0 + s_r r_m}{s_r} < \frac{1}{a_1},$$

which, in turn, is equivalent to

$$r_m < \frac{1}{a_1} - \frac{n_0}{s_r}.$$

Now using Lemma 2, we get $w^* > 0$.

Since,

$$r_m < \frac{1}{a_1} - \frac{n_0}{s_r},$$

multiplying through by $a_1 n_1 \geq 0$, gives

$$a_1 n_1 r_m \leq n_1 - \frac{a_1 n_1 n_0}{s_r},$$

which implies that

$$a_1 n_1 r_m \leq n_1 - \frac{a_1 n_1 n_0}{s_r} \leq n_1 < n_1 + a_0 n_0,$$

because $(a_0 a_1 n_1)/s_r \geq 0$ and $a_0 n_0 > 0$. Now using Lemma 1, we get $r^* > r_m$.