**Determinants of economic growth and fiscal fragility in a post-Keynesian model with public capital and targeted debt accumulation**

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**Abstract**

The question of whether a certain level of public debt makes the fiscal position more fragile and accelerates or slows economic growth is actively debated. However, empirical studies on this issue remain inconclusive, which requires more theoretical studies to identify these linkages more clearly. Therefore, we construct a post-Keynesian growth model with public capital, debt accumulation, and endogenous labour market equilibrium. Our model differs from the traditional post-Keynesian growth model in two ways. First, it incorporates a threshold or target value for the public debt ratio and constrains government spending to maintain it. Second, by extending Minsky's classification of hedge, speculative, and Ponzi financing to the government sector, our model elucidates the relationships between fiscal fragility, economic growth, and stability. The main conclusions are as follows: First, steady-state stability requires the Keynesian stability condition, the Domar stability condition, and a positive labour productivity growth rate. Second, policy coordination between the government and the central bank is important for preventing fiscal fragility in the process of high economic growth. Third, there are three cases of fiscal fragility and stability depending on the levels and changes in the saving rate, tax rate, and profit share. For high economic growth, stability, and fiscal consolidation targets, the case in which only the hedging position is stable is preferable. This case is characterised by a low saving rate, high tax rate, and low-profit share. A low saving rate and high tax rate increase the economic growth rate. A low-profit share tends to shape a wage-led growth regime and its decline further induces a higher economic growth rate. Therefore, these combinations are important determinants of the economic growth rate, stability, and sound fiscal position.

**Keywords:** post-Keynesian model, public debt, fiscal fragility

**JEL classification:** E11, E63, H63, O40

## 1 Introduction

Is public (government) debt a driver or a constraint of economic growth and stability? How do rising public debt and debt-to-GDP ratios make the fiscal status more fragile? This issue has been intensively debated in the context of fiscal austerity in the aftermath of the global financial crisis and subsequent European fiscal crisis.

In this debate, the existence of a threshold for the public debt-to-GDP ratio as a cause of economic slowdown has received particular attention. For example, Reinhart and Rogoff (2010) show that the relationship between GDP growth (economic growth rate) and the public debt ratio is non-linear and that countries with debt ratios above 90% have significantly lower average and median growth rates than those with lower ratios. In other words, an increase in the debt ratio is an obstacle to economic growth. However, after correcting for sample bias, data processing, and aggregation errors, Herdon et al. (2014) show that economic growth does not slow above the 90% threshold. They argued that the effectiveness of austerity could not be defended.

The conclusions of subsequent studies on the positive or negative association between public debt ratio and economic growth rate, and the existence of a threshold for the former vis-à-vis the latter, are not unique. Some studies show that even if a negative relationship between the two could be detected, the threshold could be at a very low proportion of the debt ratio (Égert 2015). Other studies show that there is no generally established threshold for the public debt ratio at which the economic growth rate is significantly reduced (Panizza and Presbitero 2013; Heimberger 2023). Alternatively, regardless of the threshold, it is also shown that an increase in public debt reduces both short- and long-term economic growth (Asteriou et al. 2021). Moreover, there can be a reverse causality between the two (Deformas 2015): as GDP falls, the public debt-to-GDP ratio rises, making aggressive fiscal spending more difficult. Woo and Kumar (2015) found that even after addressing these endogeneities, high public debt was significantly associated with subsequent slower growth in developed and emerging economies over the past 40 years. Taylor et al. (2012) applied a model of feedback between the primary budget deficit, the debt ratio and the economic growth rate to an econometric analysis of the US economy. They showed that an increase in the primary budget deficit has a strong positive impact on the economic growth rate.

These disputes suggest that the link between economic growth and the public t debt ratio is not the same in each country and period. It varies according to real aspects of the economy, such as production, expenditure, and income distribution. The public debt ratio can affect the magnitude of government expenditure through pressure on credit, interest payments, and fiscal discipline. Therefore, careful control of the expenditure and debt ratio in fiscal policy and of interest rate in monetary policy is important as well. With this background, neoclassical growth models have considered the effects of fiscal and debt rules to control the government's expenditure and public debt ratio on economic growth. However, post-Keynesian models that theoretically address these issues have been scarce as we will survey in the next section.

This study builds a post-Keynesian baseline model of economic growth in which labour market equilibrium and the accumulation of public capital and debt proceed simultaneously. We explicitly integrate the government's target debt ratio into the baseline model. Our model differs from previous studies in that it allows for a specific measurement of fiscal fragility during steady-state economic growth. In doing so, Minsky's classification of hedge, speculative, and Ponzi financing is a useful measure of the dynamic changes in such fiscal fragility (Minsky 1982; 1986).

Minsky (1975) explained the economic crisis in terms of private sector physical capital, debt accumulation, and an optimistic economic outlook as endogenous factors. Private firms become more fragile as they borrow to expand their businesses and earn profits, which is accelerated by an optimistic economic outlook. Financial fragility can be classified into three categories based on cash flows (Minsky 1986, Chapter 9). First, hedge finance firms can cover investment expenditures and interest payments, as well as repay part of the principal with profits. Second, in speculative finance, only interest payments can be repaid as profits. However, this is not possible in Ponzi financing. Firms with large debt and interest payments face financial difficulties, and the repayment of the principal and interest is delayed, eventually triggering an economic crisis. Big governments and central banks are expected to play a role in preventing economic crises by supporting corporate profits and employment through fiscal or monetary policies (Minsky 1986, chapters 12 and 13).

Minsky does not consider that fiscal fragility per se causes economic instability; rather, he believes that fiscal and monetary policies can prevent economic instability. Nevertheless, it is worthwhile applying Minsky's ideas in the public sector. The global financial crisis and the COVID-19 crisis required substantial fiscal spending to recover from there. Consequently, government expenditures have increased, and there has been concern that a fragile fiscal status could cause the next economic downturn. Fiscal fragility could eventually make the policy more difficult and the fiscal crisis per se may lead to future financial market turmoil, as in the European fiscal crisis triggered by Greece's fiscal problem. Debt also generates future interest payments, and the central bank anchors nominal interest rates through the monetary policy. Therefore, a high interest rate as well as the size of the outstanding debt is also crucial. Hence, a combination of the central bank's interest rate policy and the government's target debt ratio control is the key to reducing fiscal fragility. In this regard, inflation has resulted in interest rate hikes in the US and the EU that are unprecedented in recent history. As interest rates rise, the burden of public debt increases with interest payments. Japan, by contrast, accumulated a huge public debt, but historically low interest rates have mitigated the public debt burden. What are the effects of such a contrasted combination of debt and interest rate policies? Thus, asking about the implications for macroeconomic stability of public debt burden and interest rate matters.

Building a post-Keynesian model with the government's target debt ratio is also of practical importance. The Maastricht Treaty's Stability and Growth Pact restricts Euro-implementing countries to a budget deficit to GDP ratio of 3% and a public debt-to-GDP ratio of 60% as reference values to prevent and correct excessive budget deficits. In an economy with such a target debt ratio, debt financing by issuing government bonds cannot be implemented freely. Although some previous studies suggest that there is no correlation between the public debt ratio and the economic growth rate in a country, this does not mean that government finances are sustainable at any debt ratio. Implicitly or explicitly, fiscal policies are required to control the divergence of debt ratios and guarantee the creditworthiness of government bonds. If the repayment of debt is delayed and credit is undermined, this may lead to investor flights from government bonds, currency crises in the foreign exchange market, inflation, and soaring interest rates (Minsky 1986, Chapter 13, p. 336). Minsky hence emphasised the importance of proper government taxation and spending in combination with monetary policy. With such practical importance, we will analytically reveal how debt-expenditure control by government is related to economic growth and fiscal fragility.

The remainder of this paper is organised as follows: Section 2 reviews related literature and provides the novelties and contributions of this study; Section 3 briefly develops the baseline Post-Keynesian growth model; Section 4 analyses the steady state, and comprehensively identifies the determinants of economic growth, its stability and fiscal fragility; Section 4.1 confirms the existence of a long-run steady state in the model, and derives its stability conditions; Section 4.2 introduces some criteria for classifying fiscal fragility into hedge, speculative, and Ponzi fiscal positions; Section 4.3 specifically identifies the determinants of economic growth, its stability, and fiscal fragility in three cases. The implications of this study are presented in Section 4.4. Finally, section 5 concludes the paper. Note that mathematical arguments and proofs are given as far as possible to the Appendices, while the main parts are devoted to econometric discussions.

## 2 Related literature on public debt and economic growth

Regardless of the school of thought, previous research has examined how the accumulation of public debt helps or hinders long-run economic growth.

Interestingly, recent neoclassical studies consider fiscal and debt rules that constrain government spending and the public debt ratio. For example, Futagami et al (2008) is an endogenous growth model with productive government services and a target rule for the ratio of public debt to private capital. They show multiple steady states for growth rates and indeterminate transition paths. Minea and Villieu (2013) emphasise the target ratio of debt to GDP rather than the target ratio of debt to capital, which makes the transition path and the associated steady-state growth rate unique. Checherita-Westphal et al. (2014), through a neoclassical production function with public capital and empirical analysis of OECD countries, find that the optimal debt ratio that maximises the economic growth rate depends on the output elasticity of the public capital stock. Furthermore, Greiner (2012) shows that the higher the public debt ratio, the lower the long-run growth rate due to its crowding-out effect on private investment under the intertemporal budget constraint. Greiner (2013) allows for unemployment due to wage rigidities. In his model, public debt affects the stability of the economy, but long-run growth and employment are independent of it. Ono (2020) is a monetary growth model with financial intermediation, and unique in that it incorporates a public debt rule and an expenditure rule. In the former and the latter, the debt ratio and the expenditure ratio are held constant. Ono (2020) shows that a lower debt-to-GDP ratio accelerates economic growth and thus a tighter fiscal rule benefits the economy. Generally, public capital accumulation and debt financing also play a role in neoclassical growth models. However, they typically construct a supply-side growth model in which involuntary unemployment does not play a role. Moreover, economic decisions and behaviour are based on optimising payoffs. As these are assumed to be universal, they are blind to social, institutional, and structural contingencies in each country and period.

In contrast, post-Keynesian models are based on demand-led growth models and the conventional behaviour of economic agents with bounded rationality in an uncertain world. They generally support the positive effect of government expenditure and public debt on economic growth. Jong-Il and Dutt (1996) find that expansionary fiscal policies significantly increase the debt-to-GDP ratio, but increase the rate of economic growth. Dutt (2013) shows the crowding-in effect of government investment on private investment, despite the negative effect of public debt accumulation. Recent models have analysed this issue by distinguishing between government consumption and capital accumulation, and by incorporating labour market equilibrium (Tavani and Zamparelli 2017; Nishi and Okuma 2023; 2024). Tavani and Zamparelli (2017) construct a demand-led growth model with public debt, but its size is irrelevant for long-run growth. In contrast, Nishi and Okuma (2023) present a post-Keynesian model of different growth regimes with social infrastructure and public debt. However, long-run economic growth is also independent of public debt, which contradicts the empirical results below. Nishi and Okuma (2024) extend Nishi and Okuma (2023) by deriving the Domar condition, which is important for establishing a stable debt ratio in the demand-led growth process (Domar 1944; Sardoni 2024).

However, these post-Keynesian models commonly determine the public debt ratio endogenously, ignoring the government's budget constraints. In other terms, public debt is unconditionally issued whenever the government's discretionary expenditure exceeds its revenue. There is no threshold for the debt ratio, and only the so-called Domar stability condition is necessary for fiscal stability in the sense that the debt ratio converges to a constant level. Surprisingly, in contrast to neoclassical models, few post-Keynesian models have a threshold or target for the public debt ratio. How do these existences relate to economic growth and fiscal fragility?

Although post-Keynesian models on these issues are scarce, empirical studies applying the Minskian perspective to the fiscal fragility of the public sector have been presented: Ferrari-Filho et al. (2010) show that the fiscal fragility of the Brazilian public sector since 2000 has been driven by speculative fiscal positions. Furthermore, Bittes Terra and Ferrari-Filho (2021) characterise the fiscal fragility of the Brazilian government in terms of borrowing requirements and budget execution. They showed that the country fell into a Ponzi fiscal position between 2014 and 2016, although it was dominated by a speculative position after 2000. Argitis and Nikolaidi (2014) also showed that the fiscal fragility of the Greek government was behind the Greek crisis, with a series of Ponzi and ultra-Ponzi positions between 1988 and 2012. Fiscal deterioration, accompanied by large debt financing, reduced the confidence of buyers of government bonds, leading to higher interest rates on government bonds and higher interest payments. Theoretical studies in post-Keynesian models have been directed to economic growth and the private sector's financial fragility (Foley 2003; Sordi and Vercelli, 2006; Lima and Meirelles 2007; Ryoo, 2010; Charpe et al, 2011; Nishi 2012). According to Nikolaidi and Stockhammer (2017), who surveyed Minsky models, most deal with the financial fragility of the private sector. Some stock-flow consistent models explicitly include the government sector, where fiscal policy plays an important role in mitigating financial instability (Pedrosa et al. 2023). However, they do not identify fiscal fragility. Hence, it is not theoretically clear what conditions create fiscal fragility and, under these conditions, to what extent fiscal fragility is associated with economic growth and stability.

Minsky's financial fragility category of hedge, speculative, and Ponzi positions is not limited to the private sector. It is indeed effective to specify how and why fragile the government's fiscal status is in the growth process. In light of the above, this study measures fiscal fragility using hedge, speculative, and Ponzi positions, and how these relate to economic growth and stability. In doing so, we also discuss the link between the monetary and fiscal policies that control the interest rate and target debt ratio, respectively.

## 3 Model

### 3.1 Production, income distribution, and effective demand

We assume a closed economy consisting of workers, firms managed by capitalists, and the government. These economic actors try to behave rationally, but they are far from being perfectly rational in the fundamentally uncertain world. Thus, the rationale and consequences of economic actors' decisions will only be known over time. Normally, their behaviour depends mostly on convention, routine, and heuristic procedures within a particular structural setting (Skott 2023). The structural equations below assume such behaviours. Workers supply the labour force to firms and receive wages. Capitalists employ workers, earn profits from firms, and receive interest income from holding government bonds. Firms engage in productive investments and production. The government imposes the same tax rate on wages and profits, but its expenditures always exceed tax revenues and financing through government bonds. The government pays interest on outstanding government bonds, where the interest rate is set by the central bank's monetary policy. It then spends revenue from tax and government bond issues on the consumption of goods and investments in public capital. Thus, the government does not directly produce aggregate output but facilitates private firms' production through public capital provision.

Firms produce goods, which is described by the following Leontief-type fixed-coefficient production function.

$$Y = \min(uvK, qL) \tag{1}$$

where $Y$ is output, and $K$ is private capital, and $L$ is labour input. Let $\bar{Y}$ be potential output, then $v \equiv \bar{Y}/K$ denotes potential capital productivity. Additionally, $u \equiv Y/\bar{Y}$ denotes the capacity utilisation rate, which varies with effective demand. Output is determined by the operating capital stock under the effective demand constraint $Y = uvK$. When labour is efficiently demanded $Y = uvK = qL$ is realised, and accordingly the labour demand is determined by $L = uvK/q$, where $q \equiv Y/L$ is labour productivity.

Potential capital productivity $v$ is determined by the capital composition $\chi \equiv S/K$, where $S$ denotes public capital. Public capital refers to social infrastructure such as roads, communications, and public transport, which improves the potential productivity of private capital. The higher its capital composition, the more goods a firm can produce; however, its average productivity decreases. This relationship is formulated as follows:

$$v = \chi^{\sigma} \tag{2}$$

where $\sigma \in (0,1)$ is the elasticity of potential capital productivity with respect to the capital composition. If $\chi$ is zero, the economy lacks a productive foundation and cannot produce goods. A high $\chi$ increases potential capital productivity, accommodating sudden increases in effective demand. The public capital thus directly contributes to the aggregate output by enhancing capital productivity. In the long run, it also affects labour productivity growth as we will formalise below.

Gross income $pY$ is distributed to wage income $wL$ and profit income $rpK$ with a fixed share:

$$pY = wL + rpK \tag{3}$$

where $w$ is nominal wage, and $r$ is profit rate, and $p$ is price. The constant profit share $m$ is given by:

$$m = 1 - \frac{w}{pq}. \tag{4}$$

and the wage share is given by $1 - m$. The functional distribution of income is constant through the endogenous determination of the employment rate and labour productivity growth. Generally, when the employment rate rises, the labour market becomes tighter, putting upward pressure on real wages. Unless the labour productivity growth rate accelerates simultaneously, a profit squeeze occurs, reducing the profit rate for capitalists. Therefore, to maintain the profit rate, capitalists adopt labour-saving technical changes in response to an increase in the employment rate. Our model considers a situation in which an increase in the employment rate increases not only the growth rate of real wages but also the growth rate of labour productivity equally. This ensures a constant income distribution and changes in the share of the income distribution occur only through exogenous shocks.

The government imposes an equal rate of income tax $\tau \in (0,1)$ on wagthe e and profit incomes. Then, tax revenue $T$ is given by

$$T = \tau pY = \tau puvK \tag{5}$$

The government undertakes expenditures on consumption and public capital investments. Government expenditure is financed primarily by tax revenue; however, the government also relies on debt financing through the issuance of government bonds, which are subject to a certain target debt ratio. Then, the government's consumption and investment expenditures are given by $G_C = \rho_C \theta u v K$ and $G_S = \rho_S \theta u v K$, respectively. Hence, the government's total expenditure is defined as:

$$G_C + G_S = (\rho_C + \rho_S)\theta u v K \tag{6}$$

where the government's propensity to consume and invest is represented by a percentage of real income expressed by $\rho_C > 0$ and $\rho_S > 0$.

Importantly, $\theta$ denotes the government's total expenditure coefficient. This is an auxiliary variable affecting the total government expenditure, which endogenously adjusts to maintain the target debt ratio $\delta^*$ below. The formulation of the government's target debt ratio and associated government expenditures is as follows: First, new borrowing through government bonds varies according to

$$\dot{D} = p(G_C + G_S) - T + iD \tag{7}$$

where the dotted mark indicates changes in the variable over time. $D$ is the nominal debt level, $i$ is the nominal interest rate, and the debt ratio in real terms is defined as $\delta \equiv \frac{D}{pK}$. The growth rate of the debt ratio follows $\hat{\delta} = \widehat{D} - \hat{p} - \widehat{K}$, where the hat mark indicates the rate of change in the variable over time. Hence, this dynamic of the debt ratio is

$$\dot{\delta} = (\theta(\rho_C + \rho_S) - \tau)vu + (i - g)\delta \tag{8}$$

where $\widehat{K} \equiv g$ represents the actual accumulation rate, which we will formalise later. We assume that the inflation rate $\hat{p}$ is zero and the nominal interest rate $i$ is equal to the real interest rate. The government's expenditure is constrained to keep the target debt ratio constant at $\delta^*$ at a steady state $\dot{\delta} = 0$. Given the other variables, Equation (8) must ensure that the following condition for the debt-to-GDP ratio converges to $\delta^*$

$$\frac{d\dot{\delta}}{d\delta} = i - g < 0 \tag{9}$$

That is, the Domar stability condition is required and the capital accumulation rate (or the steady state economic growth rate) must be higher than the real interest rate. Currently, we impose this stability condition, and the specifics of which are analysed in Section 4. While this condition is met, the government controls the total expenditure coefficient $\theta$. Solving Equation (8) for $\theta$ at $\dot{\delta} = 0$, we obtain

$$\theta = \frac{\tau u v + (g - i)\delta^*}{(\rho_S + \rho_C)uv} \tag{10}$$

Substituting this for the government's consumption $G_C = \rho_C \theta u v K$ and investment $G_S = \rho_S \theta u v K$, respectively, we get

$$G_C = \left(\frac{\rho_C}{\rho_S + \rho_C}\right)(\tau u v + (g - i)\delta^*)K \tag{11}$$

$$G_S = \left(\frac{\rho_S}{\rho_S + \rho_C}\right)(\tau uv + (g - i)\delta^*)K \tag{12}$$

A higher target debt ratio $\delta^*$ allows for a larger fiscal deficit which in turn allows for a higher total expenditure coefficient $\theta$ and the associated greater government expenditure. Additionally, an increase in the tax rate $\tau$ generates more tax revenue, allowing for a total expenditure coefficient and thus greater government expenditure. A higher short-run economic growth rate $g$ also allows for a higher gross expenditure coefficient $\theta$. Moreover, expenditure propensities $\rho_C$ and $\rho_S$ expands the government's consumption and investment, respectively.

Workers receive wages and generate disposable income. Capitalists receive interest from profits and government bonds. For simplicity, we assume that the capitalist spends part of the disposable profit and saves all the interest income. In this case, the consumption expenditure in the private economy $C$ is as follows:

$$C = (1 - \tau)(1 - m)uvK + (1 - s)(1 - \tau)muvK \tag{13}$$

where $s \in (0,1]$ is the capitalists' saving rate from profit income.

Investment demand and the associated capital accumulation rate (the ratio of investment to capital stock) are given for the short term. Therefore, short-term investment demand $I$ is given by

$$I = gK, \tag{14}$$

where the actual accumulation rate $g$ is constant in the short term. In the long run, the demand effect of public capital increases the firms' desired rate of capital accumulation, $g_d$. Then, $g$ eventually changes with time-lag for the gestation period to achieve the desired rate of capital accumulation. We consider $g > 0$ case to obtain an analytically meaningful solution.

It follows from the above that the equilibrium in the goods market is

$$Y = C + I + G_C + G_S \tag{15}$$

Scaling this by the potential output $\bar{Y}$, and using potential capital productivity $v$, we obtain the capacity utilisation rate

$$u = \frac{(g + (g - i)\delta^*)}{sm(1 - \tau)v} \tag{16}$$

where $v$ and $g$ do not vary, and only $u$ changes only in the short run.

### 3.2 Dynamic system

Capital composition, firms' capital accumulation rates, and employment rates change in the long run, which together constitute a dynamic system.

First, the capital composition changes with public and private capital accumulation as follows.

$$\dot{\chi} = \chi(g_s - g) \tag{17}$$

where the rate of change in public capital accumulation, $g_s$ is obtained by substituting the capacity utilisation rate in

Equation (16) into Equation (12):

$$g_s \equiv \frac{\dot{S}}{S} = \frac{G_S}{S} = \left(\frac{\rho_S}{\rho_S + \rho_C}\right)\left(\frac{1}{sm(1-\tau)\chi}\right)(g\tau + (g-i)(sm(1-\tau)+\tau)\delta^*) \tag{18}$$

Adjusting the actual capital accumulation rate and the desired rate is formulated as an adaptive process. There is a so-called gestation period for investment, and the crowding-in effect of public capital on investment takes time. Therefore, capital accumulation is considered adaptive over time. The adjustment process for the capital accumulation rate is formulated as

$$\dot{g} = \kappa(g_d - g) \tag{19}$$

where $\kappa > 0$ represents a positive adjustment speed. The firm's desired rate of capital accumulation is given by the following Equation using a linearised Bhaduri and Marglin (1990) investment function:

$$g_d = \alpha + \beta m + \gamma v u \tag{20}$$

This provides the main mechanism for generating wage- and profit-led growth regimes. $\alpha$ is a constant term, the magnitude of which shall be determined by the animal spirits of the firms. $\beta$ reflects the sensitivity of investment demand to changes in the profit share. In the long run, not only $u$ varies with effective demand, but also $v$ is affected by changes in $\chi$. Therefore, $\gamma$ reflects the magnitude of the combined effect of the acceleration effect and the crowding-in effect of public capital. While acknowledging that interest rates negatively affect the desired capital accumulation rate, we discard this effect for simplicity.

As capital accumulation proceeds with realised investment, $\dot{K} = I$ is established, promoting economic growth in the long run. From Equations (19) and (20), the capital accumulation rate varies according to

$$\dot{g} = \kappa(\alpha + \beta m + \gamma v u - g) \tag{21}$$

Finally, the labour productivity growth rate is formulated using labour-saving technical changes. When this technical change is adopted, the labour productivity growth rate has a one-to-one relationship with the employment rate. Employment rate $e = \frac{L}{N}$ is defined as the ratio of labour demand $L$ and to labour supply $N$. The growth rate of labour supply is assumed to be constant at $n$. The rate of change in the employment rate is then

$$\hat{e} = \sigma(g_s - g) + g - \hat{q} - n \tag{22}$$

The labour-saving technical change explains the labour productivity growth rate as an increasing function of the employment rate in the following way:

$$\hat{q} = Ae^{\phi} \tag{23}$$

where $A$ denotes a positive constant-scale parameter. $\phi$ is the elasticity of the labour productivity growth rate with respect to the employment rate, which indicates the extent of labour-saving technical change. The acceleration in labour productivity growth rate is given by

$$\dot{\hat{q}} = \hat{q}\phi\hat{e} \tag{24}$$

The labour productivity growth rate remains constant when the employment rate is constant. Substituting (22) into (24), the acceleration of labour productivity growth rate is represented by

$$\dot{\hat{q}} = \phi\hat{q}(\sigma(g_s - g) + g - \hat{q} - n) \tag{25}$$

where the accumulation rate of public capital $g_s$ also affects the labour productivity growth rate.

## 4 Analysis

### 4.1 Existence of a steady state and determinants of economic growth

The dynamic system consists of Equations (17), (21), and (25). In this regard, we have $g_s = g_d = g$ and $g = \hat{q} + n$. There exists a non-negative stationary state $(\chi^*, g^*, \hat{q}^*)$ exists uniquely. This state is defined as

$$g^* = \frac{sm(\alpha + \beta m)(1-\tau) - \gamma i \delta^*}{sm(1-\tau) - (1+\delta^*)\gamma} \tag{26}$$

$$\hat{q}^* = g^* - n \tag{27}$$

$$\chi^* = \left(\frac{\rho_S}{\rho_S + \rho_C}\right)\left(\frac{g^*\tau + (g^* - i)(sm(1-\tau) + \tau)\delta^*}{g^* sm(1-\tau)}\right) \tag{28}$$

The capacity utilisation rate is

$$u^* = \frac{(\alpha + \beta m)(1+\delta^*) - i\delta^*}{(sm(1-\tau) - (1+\delta^*)\gamma)v(\chi^*)} \tag{29}$$

The denominators in Equations (26) and (29) are positive. Thus, the following condition was imposed:

$$sm(1-\tau) - (1+\delta)\gamma > 0 \tag{30}$$

which is the Keynesian stability condition.[1] When the labour productivity growth rate is constant, the employment rate is determined by

$$e^* = \left(\frac{1}{A}\hat{q}^*\right)^{\frac{1}{\phi}} \tag{31}$$

Thus, unemployment can persist depending on the growth rate of labour productivity. As the labour productivity growth rate is linked to the economic growth rate, an increase in the economic growth rate also increases the employment rate.[2]

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[1] The Keynesian stability condition ensures that aggregate demand changes less than aggregate supply, and excess demand decreases as capacity utilisation rates rise. If this condition is not satisfied, we cannot obtain economically meaningful solutions in the following analysis because, for example, the capacity utilisation rate, which should be positive, becomes negative.

[2] Equations (26), (27) and (31) show the two reasons why the labour market does not clear at full employment in our mode. The first reason is that capitalists adopt labour-saving technical changes in response to an increase in the

Capital productivity $v(\chi^*)$ is constant. For the local stability of this steady state, we obtain the following proposition:

**Proposition 1** Suppose the Keynesian and Domar stability conditions are satisfied. The steady state of this dynamic system is locally stable if the labour productivity growth rate $\hat{q}^*$ is non-negative.

Appendix 1: provides the proof of Proposition 1. An increase in the economic growth rate makes both the non-negative labour productivity growth rate and the domestic stability condition more likely to be satisfied. As the economic growth rate increases, so does the accumulation rate of public capital, employment rate, and labour productivity growth rate. Therefore, we shall consider the impacts of the saving rate $s$, the income tax rate $\tau$, the target debt ratio $\delta^*$, the interest rate $i$, and the profit share $m$ on the economic growth rate. The calculations are presented in Appendix 2. Note that as far as the parameters meet the stability conditions, the steady-state values are reasonable (i.e. they do not take unnecessarily negative values), and the comparative statics analysis is also economically meaningful.

A decrease in the saving rate $s$ increases the economic growth rate $g^*$. Thus, the paradox of thrift holds true. A rise in the tax rate $\tau$ increases the government expenditure and the subsequent effective demand, which in turn raises the economic growth rate.

| | $s$ | $\tau$ | $m$ | $i$ | $\delta^*$ |
|---|---|---|---|---|---|
| $g^*$ | $-$ | $+$ | $+^{PL}, -^{WL}$ | $-$ | $+$ |

**Table 1: Impacts of rising exogenous parameters on the economic growth rate**

Note: PL and WL represent profit- and wage-led growth regimes, respectively.

The impact of a change in profit share $m$ on the economic growth rate depends on the wage- and profit-led growth regimes. Profit-led growth is shaped when the profit effect and profit share are large, whereas wage-led growth is shaped when the acceleration and crowding-in effects are large, and the wage share is high. Naturally, in a profit-led growth regime, a rise in the profit share (a fall in the wage share) promotes economic growth, whereas in a wage-led growth regime, a rise in the wage share (a fall in the profit share) promotes economic growth.

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employment rate. Thus, even if a tight labour market puts upward pressure on real wages, the employment rate does not reach the full employment rate when the magnitudes of $A$ and $\phi$, which represent the effect of labour-saving technical changes in equation (31), are large. The second reason is that the model is based on the demand-led growth model. As equation (26) and Table 1 below show, economic growth is led by demand parameters, which stimulate the labour productivity growth rate in equation (27). If effective demand is insufficient, the employment rate will also fall from the full employment rate.

A lower interest rate $i$ and a higher target debt ratio $\delta^*$ also increase the economic growth rate. These changes affect not only the effective demand but also the size of the interest payment burden. Therefore, their combination is important not only as a determinant of economic growth, but also for fiscal fragility.

The effects of demand, policy, and distributional variables on economic growth rates are well known.[3] However, their relationship to fiscal fragility remains unclear. In the next section, we apply Minsky's framework of financial fragility to classify fiscal fragility.

**4.2 Definition and classification of fiscal fragility**

We define and classify fiscal fragility in terms of flows. First, it is divided into three types—hedge, speculative, and Ponzi fiscal positions—depending on how much expenditure can be covered by tax revenue in the steady state. Next, these types are presented in the $(\delta^*, i)$-plane consisting of a target debt ratio and an interest rate. The linkages between economic growth, stability, and fiscal fragility were comprehensively considered. The government controls the target debt ratio, while the central bank controls the interest rate. For example, the former's mandate may be economic policy or fiscal consolidation, while the latter's mandate may be fighting inflation or financial stabilisation.[4] As these are independent policy institutions with different mandates, their coordination is important to contain fiscal vulnerabilities while achieving stable and high economic growth.

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[3] It should be also noted that the government's propensities $\rho_C$ and $\rho_S$ do not have growth effects, but thier impacts on the capacity utilisation rate are different. A rise in $\rho_S$ has a positive level effect on the capital productivity level by inducing the government's investment in Equation (12) and the associated rise in the capital composition. It thus accommodates for a higher effective demand in Equation (16). However, a rise in $\rho_C$ has a negative level effect on the capital productivity level by reducing the capital composition. This is because a higher $\rho_C$ reduces the total expenditure coefficient $\theta$ in Equation (10) to maintain the target debt ratio, which directly decreases the government's investment. This lowers capital composition and reduces capital productivity.

[4] Monetary policy, which determines the nominal interest rate, is considered simply and exogenously in our model to focus on public debt and its impact on fiscal vulnerability and economic growth. In fact, the monetary authorities guide the interest rate to a certain level by increasing or decreasing the total volume of funds through the open market operation in the financial markets, in particular according to the credit demand of financial institutions. Although these monetary policy processes are not explicit in our model, the post-Keynesian endogenous money theory holds that the interest rate is set based on the central bank's policy instrument. Nishi (2015) provides a survey of post-Keynesian interest rate policy, and Nishi and Stockhammer (2020) analytically investigate the limits of Taylor rule interest rate policy to combat inflation and restore the level of potential output.

The classification of the government sector's fiscal fragility follows Nishi's (2012, 2019) framework of institutional sector-specific expenditures and fund revenues. Fund expenditure is defined as the sum of government spending and interest payments, whereas fund revenue is defined as the sum of tax revenues and new government bonds. Thus, an accounting Equation $T + \dot{D} = G_C + G_S + iD$ holds, which is bonded by the target debt ratio $\theta^*$. Government bond issuance in Equation (7) is based on this accounting formula.[5] On this basis, we define the criteria for financial fragility, consisting of hedge, speculative, and Ponzi fiscal positions.

A hedged fiscal position is defined as a situation in which tax revenues can finance the sum of new government bond issuances and interest payments on government bonds. Scaled by the nominal capital value, the hedge's fiscal position is given by

$$\frac{T}{pK} > \frac{\dot{D} + iD}{pK} \tag{32}$$

Speculative fiscal position occurs when tax revenues cannot cover the total new government bond issuances and interest payments, but can only cover interest payments on government bonds. In other words,

$$\frac{T}{pK} \leq \frac{\dot{D} + iD}{pK} \tag{33}$$

and

$$\frac{T}{pK} > \frac{iD}{pK} \tag{34}$$

are satisfied in the speculative fiscal position.

Ponzi fiscal position refers to a situation where even interest payments on government bonds cannot be financed by tax revenues. In other words, the government undertook

$$\frac{T}{pK} \leq \frac{iD}{pK} \tag{35}$$

in the Ponzi fiscal position.[6]

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[5] More precisely, depending on the national or regional fiscal system, expenditure includes debt redemption costs related to the principal and amortisation. These items are discarded for simplicity.

[6] Minsky (1975; 1986) defines the safety margin $\mu$ as the ratio of the present value of the return on capital assets to the present value of debt principal and interest payments, classifying the financial fragility of firms into three categories. Earnings, principal, and interest payments fluctuate with economic activity and the discount rate varies from firm to firm over time. This is also true for government revenues and expenditure. It is therefore difficult to theoretically classify them into hedging, speculative and Ponzi positions in a strict way, hence our classification depends on the current value in each

The tax revenue $T$ and government bond issuance $\dot{D}$ appearing in these Equations are given by Equations (5) and (7), respectively, in the steady state. For the Keynesian stability condition in Equation (30) to be satisfied, the range of the target debt ratio must satisfy

$$\delta^* < \delta_{\mathrm{K}} \equiv \frac{sm(1-\tau)-\gamma}{\gamma} \tag{36}$$

This represents a threshold for a stable quantity adjustment, which depends on the country's investment-saving parameters. In addition, the Domar stability condition must be satisfied in a steady state. This condition is defined as follows:

$$g^* - i = \frac{sm(\alpha+\beta m)(1-\tau) - (sm(1-\tau)-\gamma)i}{sm(1-\tau)-(1+\delta^*)\gamma} > 0 \tag{37}$$

Solving this in the range of $\delta^* < \delta_{\mathrm{K}}$,

$$i < i_{\mathrm{K}} \equiv \frac{sm(\alpha+\beta m)(1-\tau)}{sm(1-\tau)-\gamma} \tag{38}$$

must be satisfied. From these arguments, we obtain the following proposition.

**Proposition 2** The Domar stability condition is satisfied when the actual interest rate is set lower than the lowest interest rate $i_{\mathrm{K}}$ in the range of the target debt ratio that satisfies the Keynesian stability condition.

These ranges specifically categorise the domains in which hedge, speculative, and Ponzi fiscal positions are realised.

### 4.2.1 Hedge and speculative fiscal positions

The domain for hedging and speculative fiscal positions can be divided from Equation (32) as

$$i < i_{hs}(\delta^*) \equiv \frac{(\alpha+\beta m)\left(\tau+\delta^*\big(\tau - sm(1-\tau)\big)\right)}{\delta^*(sm(1-\tau)+\tau-\gamma-2\gamma\delta^*)} \tag{39}$$

where $\tau > sm(1-\tau)$ is assumed for the numerator to be positive. This implies that government tax revenues are greater than the after-tax savings of capitalists.[7] In addition, we assume $sm(1-\tau)+\tau-\gamma > 2\gamma\delta^*$ for the denominator to be positive. As the nominal interest rate is non-negative, the denominator of Equation (39) is positive.

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period.

[7] When this condition is not fulfilled, there is no longer a positive interest rate that satisfies the hedge fiscal position when the target debt ratio exceeds a certain ratio.

Therefore,

$$\delta^* < \delta_{hs} \equiv \frac{sm(1-\tau)+\tau-\gamma}{2\gamma} \tag{40}$$

is necessary. As $\delta_K < \delta_{hs}$ always holds, the range of target debt ratio that satisfies Keynesian stability is the binding condition. Moreover, at $\delta^* = \delta_K$

$$i_{hs}(\delta_K) = \frac{sm(\alpha+\beta m)(1-\tau)}{sm(1-\tau)-\gamma} \equiv i_K \tag{41}$$

is obtained.

We characterise the shape of the graph that separates hedge and speculative fiscal positions in the $(\delta^*, i)$-plane. Differentiating $i_{hs}(\delta^*)$ with $\delta^*$, we have

$$\frac{di_{hs}(\delta^*)}{d\delta^*} = \frac{(\alpha+\beta m)}{\delta^{*2}(sm(1-\tau)+\tau-\gamma-2\gamma\delta^*)^2} H(\delta^*), \tag{42}$$

where $H(\delta^*) \equiv a_h\delta^{*2} + b_h\delta^* + c_h$; $a_h \equiv 2\gamma\big(\tau - sm(1-\tau)\big) > 0$, $b_h \equiv 4\gamma\tau > 0$, and $c_h \equiv -(sm(1-\tau)+\tau-\gamma)\tau < 0$. Therefore, there exists one positive $\delta^*$ that satisfies $H(\delta^*) = 0$. Let this be $\tilde{\delta}_{hs}$. Then, $i_{hs}(\delta^*)$ is a downturn curve for $0 < \delta^* < \tilde{\delta}_{hs}$ and an upturn curve for $\tilde{\delta}_{hs} < \delta^*$ in the $(\delta^*, i)$-plane. Also, the asymptote of the $i_{hs}(\delta^*)$-curves are $\delta^* = 0$ and $\delta^* = \delta_{hs}$. It follows that the domain of hedge fiscal position exists in $\delta^* < \delta_K$ and $i < i_{hs}(\delta^*)$. The domain of speculative and Ponzi fiscal position exists in $\delta^* < \delta_K$ and $i > i_{hs}(\delta^*)$.

### 4.2.2 Speculative and Ponzi fiscal positions

The domain between the speculative and Ponzi fiscal positions can be divided from Equations (34) and (35) as follows:

$$i < i_{sp}(\delta^*) \equiv \frac{(\alpha+\beta m)(1+\delta^*)\tau}{\delta^*\big(sm(1-\tau)+\tau-\gamma(1+\delta^*)\big)} \tag{43}$$

As the nominal interest rate is positive, for this denominator

$$\delta^* < \delta_{sp} \equiv \frac{sm(1-\tau)+\tau-\gamma}{\gamma} \tag{44}$$

is required. As $\delta_K < \delta_{sp}$ always holds, the range of target debt ratio that satisfies Keynesian stability is the binding condition. Moreover, comparing Equations (40) and (44) shows that $\delta_{hs} < \delta_{sp}$ is established.

Moreover, at $\delta^* = \delta_K$

$$i_{sp}(\delta_K) = \frac{sm(\alpha+\beta m)(1-\tau)}{sm(1-\tau)-\gamma} \equiv i_K \tag{45}$$

is established. That is, the curves for $i_{sp}(\delta^*)$ and $i_{hs}(\delta^*)$ have an intersection coordinate at $\delta^* = \delta_K$.

We characterise the shape of the graph separating the speculative and Ponzi fiscal positions in the $(\delta^*, i)$-plane. Differentiating $i_{sp}(\delta^*)$ with $\delta^*$, we have

$$\frac{di_{sp}(\delta^*)}{d\delta^*} = \frac{(\alpha + \beta m)\tau}{\delta^{*2}(sm(1-\tau) + \tau - \gamma(1+\delta^*))^2} S(\delta^*), \tag{46}$$

where $S(\delta^*) \equiv a_s\delta^{*2} + b_s\delta^* + c_s$ ; $a_s \equiv \gamma > 0$ , $b_s \equiv 2\gamma > 0$ , and $c_s \equiv -(sm(1-\tau) + \tau - \gamma) < 0$ and since$S(\delta^*) = 0$ Therefore, there exists one positive $\delta^*$ that satisfies $S(\delta^*) = 0$. Let this be $\tilde{\delta}_{sp}$. Then, $i_{sp}(\delta^*)$ is a downturn curve for $0 < \delta^* < \tilde{\delta}_{sp}$ and is an upturn curve for $\tilde{\delta}_{sp} < \delta^*$ in the $(\delta^*, i)$-plane. Also, the asymptotes of the $i_{sp}(\delta^*)$-curves are $\delta^* = 0$ and $\delta^* = \delta_{sp}$. It follows that the domain of speculative fiscal position exists in $\delta^* < \delta_{\mathrm{K}}$ and $i_{hs}(\delta^*) < i < i_{sp}(\delta^*)$, whereas the domain of Ponzi finances exists in $\delta^* < \delta_{\mathrm{K}}$ and $i_{sp}(\delta^*) < i$.

### 4.2.3 Economic growth in the stable domain of target debt ratio and interest rate

To classify fiscal fragility and link it to economic growth, economically meaningful steady states must be considered. In particular, capacity utilisation and economic growth rates must always be positive. Therefore, for the numerator of the capacity utilisation rate Equation (29), $(\alpha + \beta m)(1 + \delta) - i\delta^* > 0$ is necessary. Hence, the interest rate must satisfy the following conditions.

$$i < i_u(\delta^*) \equiv \frac{(\alpha + \beta m)(1 + \delta^*)}{\delta^*} \tag{47}$$

The $i_u(\delta^*)$-curve is a downturn curve in $\delta^* < \delta_{\mathrm{K}}$, and more bounding than the condition for a positive numerator in the economic growth rate in Equation (26). It is also clear that at $\delta^* = \delta_{\mathrm{K}}$

$$i_u(\delta_{\mathrm{K}}) = \frac{sm(\alpha + \beta m)(1-\tau)}{sm(1-\tau) - \gamma} \equiv i_{\mathrm{K}} \tag{48}$$

is valid.

From Equation (26), the iso-economic growth rate $\tilde{g}$ can be drawn in the $(\delta^*, i)$-plane, given the parameters other than $\delta^*$ and $i$. This can be expressed as

$$i = i_{\tilde{g}}(\delta^*) \equiv \frac{sm(\alpha + \beta m)(1-\tau) - \tilde{g}\big(sm(1-\tau) - \gamma(1+\delta^*)\big)}{\gamma\delta^*} \tag{49}$$

which we call the iso-growth rate curve. As the iso-growth rate curve satisfies $\frac{\partial i_{\tilde{g}}(\delta^*)}{\partial \delta^*} > 0$ and $\frac{\partial i_{\tilde{g}}^2(\delta^*)}{\partial \delta^{*2}} < 0$ for $\tilde{g} > 0$, the curve is upturn in the $(\delta^*, i)$-plane. In other words, for realising a certain economic growth rate, the target debt ratio must also be low, when the interest rate is low, and conversely, the target debt ratio must also be high when the interest rate is high.[8] Additionally, because a stable steady state is considered, the labour productivity growth rate must be positive and $\tilde{g} > n$ must always be satisfied. The x-intercept of the $i_{\tilde{g}}(\delta^*)$-curve is

$$\delta_{\tilde{g}} = \frac{\tilde{g}(sm(1-\tau)-\gamma) - sm(\alpha+\beta m)(1-\tau)}{\tilde{g}\gamma} \tag{50}$$

where $\frac{d\delta_{\tilde{g}}}{d\tilde{g}} > 0$ holds. That is, given a certain stable domain, the x-intercept of the $i_{\tilde{g}}(\delta^*)$-curve representing the higher economic growth rate is in a more rightward position.

The impacts of saving rate $s$, tax rate $\tau$, and income distribution $m$ for this intercept $\delta_{\tilde{g}}$, evaluated at the steady-state economic growth rate are respectively given as follows

$$\frac{\partial \delta_{\tilde{g}}}{\partial s} = \frac{m\big((\alpha+\beta m) + (\alpha+\beta m - i)\delta^*\big)(1-\tau)}{sm(\alpha+\beta m)(1-\tau) - \gamma i \delta^*} > 0 \tag{51}$$

$$\frac{\partial \delta_{\tilde{g}}}{\partial \tau} = -\frac{m\big((\alpha+\beta m) + (\alpha+\beta m - i)\delta^*\big)}{sm(\alpha+\beta m)(1-\tau) - \gamma i \delta^*} < 0 \tag{52}$$

$$\frac{\partial \delta_{\tilde{g}}}{\partial m} = -\frac{s(1-\tau)G(m)}{\gamma(sm(\alpha+\beta m)(1-\tau) - \gamma i \delta^*)} \lesseqgtr 0 \tag{53}$$

where $i = 0$. In a new stable domain formed by a lower saving rate and a higher tax rate, the x-intercept of the curve represents the same economic growth rate as before the change and is in a more leftward position. The sign of $G(m)$ is either positive or negative, depending on the profit-led and wage-led growth regimes, respectively. Therefore, an increase in profit share in the former and an increase in wage share in the latter shift the x-intercept of the $i_{\tilde{g}}(\delta^*)$-curve to a leftward position, representing the same economic growth rate as before the change in the new stable domain.

Furthermore, the iso-growth rate curve satisfies

$$i_{\tilde{g}}(\delta_{\mathrm{K}}) = \frac{sm(\alpha+\beta m)(1-\tau)}{sm(1-\tau)-\gamma} \equiv i_{\mathrm{K}} \tag{54}$$

Hence, all curves for $i_u(\delta^*)$, $i_{hs}(\delta^*)$, $i_{sp}(\delta^*)$ and $i_{\tilde{g}}(\delta^*)$ have a unique intersection at $\delta^* = \delta_{\mathrm{K}}$.

We now summarise this argument in Proposition 3.

**Proposition 3** Given other investment-saving parameters, $0 < i < i_{\mathrm{K}}$ and $0 < \delta^* < \delta_{\mathrm{K}}$ form stable domain and associated fiscal fragility in the $(\delta^*, i)$-plane. Of these, the hedge fiscal domain is $0 < i < i_{hs}(\delta^*)$, the speculative fiscal domain is $i_{hs}(\delta^*) \leq i < i_{sp}(\delta^*)$, and the Ponzi fiscal domain is $i_{sp}(\delta^*) \leq i$.

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[8] Which iso-growth rate curve is realised naturally depends on all exogenous variables. However, given the parameters related to the investment and saving functions and the tax rate (i.e. $\alpha, \beta, \gamma, s, m,$ and $\tau$) are given, the realisation of the economic growth rate ultimately depends on the combination of the interest rate and the target debt ratio.

The positions of each curve, the associated interest rate $i_K$ and target debt ratio $\delta_K$ are affected by changes in the saving rate, tax rate and income distribution (Table 2). An increase in the saving rate and a decrease in the tax rate reduce the interest rate and increase the target debt ratio. An increase in profit share raises the target debt ratio, satisfying Keynesian stability; however, there are two ways to impact the interest rate. In a wage-led regime, the interest rate may rise or fall. However, in a profit-led growth regime, the interest rate always rises. This led to a change in the range of the iso-growth rate curve. Proofs are provided in Appendix 3.

### 4.3 Determinants of fiscal fragility

#### 4.3.1 Three cases of financial vulnerability

The $i_{hs}(\delta^*)$- and $i_{sp}(\delta^*)$-curves have $\tilde{\delta}_{hs}$ and $\tilde{\delta}_{sp}$ on the x-axis, respectively, and are parabolas convex downwards curves in the $(\delta^*, i)$-plane. According to the slopes of these curves at $\delta_K$, there are three cases for the combination of financial fragility and steady state stability. Which case is realised depends on all the exogenous variables, which means that the government cannot fully control fiscal fragility using only its policy parameters. Here, we specify these three cases based on the profit share $m$, the tax rate $\tau$, and the saving rate $s$.[9] The results are summarised in Table 2. The proofs of the existence of each case are provided in Appendix 4.

| | Fiscal fragility and steady state stability | | | | | |
|---|---|---|---|---|---|---|
| | Case 1. All HSPs are stable. | Case 2. HS stable, P unstable. | Case 3. H stable, SP unstable. | $i_K$ | $\delta_K$ | $g^*$ |
| $s$ | $\tilde{s}_{sp}^{+} < s$ | $\tilde{s}_{hs}^{+} < s < \tilde{s}_{sp}^{+}$ | $s < \tilde{s}_{hs}^{+}$ | $-$ | $+$ | $-$ |
| $\tau$ | $\tau < \tilde{\tau}_{sp}^{+}$ | $\tilde{\tau}_{sp}^{+} < \tau < \tilde{\tau}_{sh}^{+}$ | $\tilde{\tau}_{hs}^{+} < \tau$ | $+$ | $-$ | $+$ |
| $m$ | $\tilde{m}_{sp}^{+} < m$ | $\tilde{m}_{hs}^{+} < m < \tilde{m}_{sp}^{+}$ | $m < \tilde{m}_{hs}^{+}$ | $\pm$ | $+$ | $\pm$ |

**Table 2: Impact of rising exogenous parameters on fiscal fragility**

Note: We assume $s_K < s$, $m_K < m$, and $\tau < \tau_K$ for each impact. Where H represents hedge, S represents speculative, and P represents Ponzi fiscal positions. See also Appendix 4.

[9] It is also possible to define a plane consisting of parameters such as the profit share and the saving rate, given other parameters than the interest rate and the debt ratio. However, to focus on central bank and government policies, this study approaches in terms of $(\delta^*, i)$-plane. In addition, the parameters of the investment function $\alpha$, $\beta$, and $\gamma$ are also concerned with on the curves' positions, but to avoid complicating the issue, these parameters are considered as given.

The reason why there are three cases of fiscal fragility in the stable domain depending on the saving rate, tax rate, and profit share relates to the fact that the $i_{hs}(\delta^*)$ and $i_{sp}(\delta^*)$- curves are parabolas convex downwards and changes in the target debt ratio satisfying the Keynesian stability condition. The reason these curves are parabolic convex downwards can be explained as follows: A rise in the saving rate and the profit share or a reduction in the tax rate increases the target debt ratio satisfying the Keynesian stability condition $\delta_K$. When the target debt ratio $\delta^*$ increases, the associated borrowing and interest payments initially increase much more than tax revenue per nominal capital. Then, interest rate $i$ must be gradually lowered to reduce interest payments and maintain fiscal fragility within the hedge or speculative range; however, as the target debt ratio rises, tax revenues will eventually increase significantly through the demand effect of increased government spending. This generates a domain in which fiscal fragility can be controlled even if the interest rate is raised.[10] Therefore, as the debt ratio satisfying the Keynesian stability condition increases, the interest rate satisfying the Domar stability condition also increases, forming a stable speculative fiscal domain, followed by a Ponzi fiscal domain.

In the following sections, the link between fiscal fragility and economic growth and stability in these three cases is analysed in detail.

#### 4.3.2 Case 1: Hedge, speculative. and Ponzi fiscal positions are all stable

In Case 1, there are stable areas for all steady states: hedge, speculative, and Ponzi fiscal positions. In this case, the steady state can be maintained even when the fiscal position is fragile. This case occurs when the profit share and saving rate are relatively high or when the tax rate is low.

Insert Figure 1 about here.

In this case, we consider the link between fiscal fragility and economic growth. If both the interest rate and target debt ratio are low, the fiscal position is stable with hedged finances. In this case, the economic growth rate is low (A), as indicated by the low iso-growth curve $\tilde{g}_L$. If the central bank raises the interest rate and the government sets a

[10] For example, the discriminant equation (34) for speculative position in steady state is given by $\tau u^*(\delta^*)v > i\delta^*$. The right-hand side is the interest payment, which monotonically increases with $\delta^*$. Moreover, $u^*(\delta^*)$ in the left-hand side is an increasing function of the debt ratio $\delta^*$, which gradually increases tax revenues owing to the demand effect of the increase in this ratio. The discriminant equation (32) for hedge position can be written as $\tau u^* v - g^* > i\delta$ which also contains a similar mechanism. However, a lower rate of interest is required to maintain this position, to the extent that the economic growth rate $g^*$ is also added.

high target debt ratio, the economy moves to a speculative or Ponzi fiscal position (from A to B) while maintaining a low economic growth rate. Meanwhile, the combination of a low interest rate and high target debt ratio may achieve a high economic growth rate as $\tilde{g}_H$-curve shows a stable hedged fiscal position (C). If the interest rate and target debt ratio increase when a high economic growth rate is maintained, the fiscal position quickly becomes more speculative (from C to D) and possibly more fragile to the Ponzi fiscal position compared with a case where a low economic growth rate is maintained.

#### 4.3.3 Case 2: Hedge and speculative fiscal positions are stable, but the Ponzi fiscal position is unstable

In Case 2, there is a stable domain for the hedge and speculative fiscal positions, but no stable domain for the Ponzi fiscal position. This occurs at intermediate profit share, saving rate, and tax rate.

Insert Figure 2 about here.

In this case, as the $\tilde{g}_L$- and $\tilde{g}_H$-curves show that both hedge and speculative fiscal positions are stable irrespective of the economic growth rates. If the interest rate and target debt ratio are low, the government's fiscal position is stable with a hedge fiscal position; however, as the $\tilde{g}_L$-curve shows, the economic growth rate is low (A). When the economy grows at the same rate, higher interest rates and target debt ratios are more likely to establish a speculative fiscal position (B). In this case, when the economy maintains a higher rate of economic growth, an increase in the interest rate weakens the speculative fiscal position more quickly than when it maintains a lower rate of economic growth (from C to D).

#### 4.3.4 Case 3: Hedge fiscal position is stable, but speculative and Ponzi fiscal positions are unstable

In Case 3, only the hedge fiscal position is in the stable domain, whereas the speculative and Ponzi fiscal positions are always unstable. In other words, weak statuses, such as speculative and Ponzi fiscal positions, are equivalent to unstable steady states. This occurs when the profit share and saving rate are relatively low or when the tax rate is high.

Insert Figure 3 about here.

As points A, B, C, and D on the $\tilde{g}_L$- and $\tilde{g}_H$ curves indicate, the stable domain includes only the hedge fiscal position, regardless of the economic growth rate, interest rate, or target debt ratio. However, as observed in Table 2, the target debt ratio satisfying the Keynesian stable domain is the smallest compared with the other two cases. Increasing the interest rate and target debt ratio, while maintaining high economic growth rates, does not lead to a weaker fiscal position within the stable domain. However, speculative or Ponzi fiscal positions do not satisfy the Domar stability condition, implying that an excessive increase in interest rates leads to immediate destabilisation.[11]

### 4.4 Determinants of economic growth, stability, and fiscal fragility

We have specified the relationship between the economic growth rate, its stability, and fiscal fragility. Given the other parameters constituting the steady state, a stable domain is formed in the $(\delta^*, i)$-plane, on which the curves demarcating the hedge, speculative and Ponzi fiscal positions are inserted. There are three cases for overlap between these curves and the stable domain, depending on the saving rate, tax rate, and profit share. Given the interest rate and target debt ratio in this domain, economic growth rate, stability, and fiscal fragility are determined. We derived the following implications from the above analysis.

First, a high target debt per se ratio does not necessarily cause low economic growth, instability, or fiscal fragility. They also depend on interest rates. Consider the case in which three fiscal positions exist in the stable domain, as in Case 1. When the target debt ratio is relatively high, but the interest rate is low, a hedge fiscal position can be achieved, and fiscal fragility can be prevented (C in Figure 1). In other words, the view that a high target debt ratio implies a fragile fiscal status, and hence, low growth, is one-sided. Its fragility and link to economic growth must also be discussed in light of interest rates. Hence, the policy co-ordination of the government and the central bank towards these objectives matters. Interest rate policy plays a supporting role not only in economic growth but also in public debt management.

Second, the link between economic growth, stability, and fiscal fragility does not depend solely on target debt ratios or interest rates. As Table 2 shows, the linkages also depend on the saving rate, tax rate, and profit share, depending on the range within which they fall.

For example, when the saving rate is high ($\tilde{s}_{sp}^{+} < s$), a stable domain is formed, including not only hedging fiscal positions but also speculative and Ponzi fiscal positions (Case 1). When the saving rate falls to $\tilde{s}_{hs}^{+} < s < \tilde{s}_{sp}^{+}$, the Ponzi fiscal position first disappears from the stable domain (Case 2), and a further decrease to $s < \tilde{s}_{hs}^{+}$ realises only the hedge fiscal position in the stable domain (Case 3). Figure 4 depicts the shift from Case 1 to Case 3 owing to a significant decrease in the saving rate from $\tilde{s}_{sp}^{+} < s$ to $s < \tilde{s}_{hs}^{+}$.

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[11] As shown in Equation (9), the Domar stability condition ensures that the actual debt ratio converges stably to the target ratio. Thus, the destabilisation here means that the actual debt ratio diverges. Alternatively, the primary balance per nominal capital in steady state is given by $(T - G_C - G_S)/pK = -(g - i)\delta^*$. The failure to satisfy the Domar stability condition therefore corresponds also to wasteful spending case where the primary balance is persistently in surplus, but the government issues bonds and pays interest on them.

The stable domain in Case 1 was formed by a domain containing $S_1$. In this case, all the fiscal statuses are compatible with the stability conditions. The iso-growth rate curve of an economy passing over $S_1$ realises a hedge position at position A, a speculative position at position B, and a Ponzi position at position C. If the economy moves to Case 3, owing to a reduction in the saving rate above a threshold, a stable domain containing $S_3$ is formed, where the stable interest rate rises, and the stable target debt ratio falls. Thus, B and C do not satisfy the Keynesian stability condition. In addition, myriad new iso-growth rate curves passing through $S_3$ can be drawn. Of these curves, the curve showing a higher economic growth rate $\tilde{g}_H$ will always have a larger x-intercept than the original curve, also passing through the coordinates of the target debt ratio and interest rate on the original curve. For example, if the same interest rate and target debt ratio are maintained as in A, stability conditions are maintained, while the economic growth rate increases.[12] The same picture is depicted for the change from lower to higher tax rates. Thus, lower saving rate and higher tax rate ensure higher economic growth and stability, while excluding the possibility of a fragile fiscal status, such as speculative and Ponzi positions.

Third, different results for changes in profit share are obtained for different growth regimes. A decrease in the profit share (increase in the wage share) in a wage-led growth regime is illustrated in Figure 5, and an increase in the profit share in a profit-led growth regime is illustrated in Figure 6.

Insert Figure 5 about here.

A wage-led growth regime is more likely to be established with a low profit share. Low-profit share also facilitated the formation of Case 3. A higher economic growth rate is achieved when the profit share is even lower in this regime. In other words, a lower profit share in a wage-led growth regime accelerates economic growth while achieving only a stable hedge fiscal position. In this process, the new stable domain shifts from the $S_W$ to the domain containing the $S_{WW}$, through which the new iso-growth rate curve passes. The target debt ratio forming the stable domain falls as

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[12] The x-intercept of the iso-growth rate curve after the change is located to the left of the original one in the new stable domain. That is, the curve with the x-intercept more to the right of the iso-growth rate curve after the change with respect to it shows a higher growth rate. It is together with Equation (A10) that this curve can further pass through the original coordinates, as at A. Equation (A10) shows that a decrease in the saving rate leads to a higher economic growth rate even if the debt ratio and interest rate remain unchanged. However, it is not possible to identify whether the new iso-growth rate curve passes through hedge position, as in A, or through other positions, as it depends on the parameters that change and how high the equal growth rate curve is drawn under these conditions. This also applies to Figures 5 and 6.

Insert Figure 4 about here.

the interest rate rises; therefore, the room for fiscal policy is smaller and that for monetary policy is larger. After the shift, B becomes unstable, while C achieves stable and high growth while maintaining its hedge fiscal position. Even if the same interest rate and target debt ratio are maintained (e.g. A), a higher rate of economic growth can be achieved through the growth effect of an increase in the wage share.

A profit-led growth regime is more likely to form with a high profit share. A high profit share realises stability not only in hedged fiscal positions but also in speculative and even Ponzi fiscal positions (Case 1). If the high profit share increases further, the new stable domain shifts from the domain with an $\mathrm{S_P}$ to the domain with an $\mathrm{S_{PP}}$, and both fiscal and monetary policies become more flexible. If the initial interest rate and target debt ratio are maintained after an increase in profit share, higher growth is achieved. Figure 6 depicts a case where high growth is achieved while the speculative fiscal position is maintained at B. However, hedge finance can continue to be maintained at low interest rates and target debt ratio, as near A. Moreover, C is unstable before the change in the profit share, but after the change, stable and high growth is achieved while involving the Ponzi fiscal position. Thus, an increase in profit share in a profit-led growth regime accelerates economic growth, while also involving fragile fiscal positions.[13]

Insert Figure 6 about here.

Of the three cases, a higher economic growth rate and stabilisation should occur in Case 3 in light of fiscal consolidation. Case 3 can be formed when the profit share and saving rate are low or when the tax rate is high. In this case, only the hedge fiscal position exists in the stable domain. Compared to the other cases, Case 3 allows for a higher stable interest rate but requires a lower stable target debt ratio, meaning that the fiscal policy is less flexible. Despite fiscal constraints, the lower saving and tax rates in Case 3 simultaneously deliver a higher economic growth rate, stability, and sound fiscal position. A wage-led growth regime is also more likely to form in Case 3, in which an increase in the wage share has the same effect. These are crucial determinants of economic growth, stability, and fiscal health.

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[13] Changes in the income distribution may simultaneously change the types of growth regime and the fiscal fragility. When the profit share increases in a wage-led growth regime, speculative (Case 2) and Ponzi (Case 1) positions gradually appear in the stable domain. Conversely, as the profit share decreases in a profit-led growth regime, the Ponzi (Case 2) and speculative (Case 3) positions gradually disappear from the stable domain and only the hedge position realises the stable domain. It is unclear whether respective growth regimes are maintained in this process. This is because, as shown in Appendix 2, a change in the profit share may transform one growth regime into the other. The compatibility of each case with the growth regime can be investigated by analysing the positional relationship between $S_K(m)$, $H_K(m)$, and $G(m)$. However, nothing analytically definitive can be identified about this relationship.

A lower saving rate and a higher tax rate in Case 1 also lead to higher growth, but this may induce more fiscal fragility. A profit-led growth regime is more likely to form in Case 1, in which an increase in the profit share has the same effect on high growth, stability, and fiscal fragility. Even in the stable domain, the persistence of the Ponzi or a speculative fiscal position, in particular, may induce risks in the future, such as a decline in the creditworthiness of government bonds, which can eventually be an obstacle to fiscal consolidation and economic growth. Therefore, it is not desirable. Case 2 is an intermediate case between Cases 1 and 3.

**5 Conclusion**

Although the relationship between economic growth and the size of the government's target debt ratio has attracted attention, its association with fiscal fragility and interest rate setting has not yet been theoretically elucidated. Therefore, applying Minsky's ideas, we classified fiscal fragility into hedge, speculative, and Ponzi positions and examined their relationship with economic growth and stability. The main conclusions are as follows:

First, a unique nonnegative steady state exists in the model. The stability of this state requires a Keynesian stability condition, Domar stability condition, and positive labour productivity growth rate. A high tax rate and low saving rate contribute to high growth rates and stability. They also discourage fragile fiscal positions such as speculative and Ponzi positions. A higher target debt ratio and lower interest rates increase the economic growth rate. However, the higher the debt ratio and interest rate, the more fragile the fiscal position is. In light of this, the recent interest rate hike due to inflation may hurt the economic growth and fiscal position, also violating the Domar condition if it goes excessively. Besides, a lower profit share in a wage-led growth regime is likely to accelerate economic growth and ensure a stable hedging position. However, an increase in profit share in a profit-led growth regime is likely to realise a higher growth rate while maintaining fragile fiscal positions.

Second, high debt ratio and fiscal fragility do not immediately imply economic instability or low growth rates. The determination of fragility and its coexistence with a stable steady state depends on the saving rate, tax rate, profit share, interest rate, and target debt ratio. The monetary policy is authorised by the central bank, whereas the fiscal policy is determined by the government, however, these two institutions are principally different and independent. Thus, no mechanism automatically links different decisions, although policy co-ordination between the government and central bank is important for preventing fiscal fragility in the economic growth process.[14]

Third, depending on the level and changes in the saving rate, tax rate, and profit share, three cases exist for

[14] The issue of coordination is more complicated in countries such as the euro area, where debt cannot be issued in the country's own currency. Moreover, the monetary and fiscal authorities are different in these areas, making cooperation even more important.

fiscal fragility and stability. In Case 1, the hedge, speculative, and Ponzi positions are compatible with a stable steady state. In Case 2, the hedge and speculative positions are compatible with a stable steady state, whereas the Ponzi position is unstable. In Case 3, only the hedge position is compatible with the stable steady state, whereas the speculative and Ponzi positions are unstable. To achieve high growth, stability, and fiscal consolidation, Case 3, in which only the hedge position is stable, is preferable. This case is established by a low saving rate, high tax rate, and low profit share. A low saving rate and high tax rate increase the economic growth rate. A low profit share shapes a wage-led growth regime, and its decline further induces a higher economic growth rate. A high economic growth rate also contributes to stability because it promotes a positive labour productivity growth rate. Therefore, these combinations are important determinants of the economic growth rate, stability, and robust fiscal position. The other cases involve speculative and Ponzi positions within the stable domain. Hence, even if some change promotes high growth and stability, a fragile fiscal position may be at risk.

Our model focuses on the steady-state by analysing the relationships between fiscal fragility, economic growth, and stability. These relationships can vary with the transitional dynamics. Therefore, further studies are required to understand the endogenous processes through which stability creates instability. The monetary authority is also responsible for preventing such instability. Hence, an explicit modelling of the central bank's monetary policy process to determine the interest rate, which we considered as exogenous, should be analytically specified. These are the remaining issues in this study.

**References**


Argitis, G., Nikolaidi, M., 2014. The financial fragility and the crisis of the Greek government sector. International Review of Applied Economics 28, 274-292.

Asteriou, D., Pilbeam, K., Pratiwi, C.E., 2021. Public debt and economic growth: panel data evidence for Asian countries. Journal of Economics and Finance 45, 270-287.

Bhaduri, A., Marglin, S., 1990. Unemployment and the real wage: the economic basis for contesting political ideologies. Cambridge Journal of Economics 14, 375-393.

Bittes Terra, F.H., Ferrari-Filho, F., 2021. Public Sector Financial Fragility Index: an analysis of the Brazilian federal government from 2000 to 2016. Journal of Post Keynesian Economics 44, 365-389.

Charpe, M., Flaschel, P., Hartmann, F., Proaño, C., 2011. Stabilizing an unstable economy: fiscal and monetary policy, stocks, and the term structure of interest rates. Economic Modelling 28, 2129-2136.

Checherita-Westphal, C., Hughes Hallett, A., Rother, P., 2014. Fiscal sustainability using growth-maximizing debt targets. Applied Economics 46, 638-647.

Domar, E.D., 1944. The burden of the debt" and the national income. American Economic Review 34, 798-827.

Dutt, A.K., 2013. Government spending, aggregate demand, and economic growth. Review of Keynesian Economics 1, 105-119.

Ferrari-Filho, F., Terra, F.H.B., Conceição, O.A.C., 2010. The financial fragility hypothesis applied to the public sector: an analysis for Brazil's economy from 2000 to 2008. Journal of Post Keynesian Economics 33, 151-168.

Foley, D.K., 2003. Financial fragility in developing economies. In: Dutt, A.K., Ros, J. (Eds.). Development Economics and Structuralist Macroeconomics: Essays in Honour of Lance Taylor. Cheltenham, UK and Northampton: Edward Elgar Publishing, 157-168.

Futagami, K., Iwaisako, T., Ohdoi, R., 2008. Debt policy rule, productive government spending, and multiple growth paths. Macroeconomic Dynamics 12, 445-462.

Greiner, A., 2012. Public debt in a basic endogenous growth model. Economic Modelling 29, 1344-1348.

Greiner, A., 2013. Sustainable public debt and economic growth under wage rigidity. Metroeconomica 64, 272-292.

Heimberger, P., 2023. Do higher public debt levels reduce economic growth? Journal of Economic Surveys 37, 1061-1089.

Herndon, T., Ash, M., Pollin, R., 2014. Does high public debt consistently stifle economic growth? A critique of Reinhart and Rogoff. Cambridge Journal of Economics 38, 257-279.

Lima, G.T., Meirelles, A.J.A., 2007. Macrodynamics of debt regimes, financial instability, and growth. Cambridge Journal of Economics 31, 563-580.

Minea, A., Villieu, P., 2013. Debt policy rule, productive government spending, and multiple growth paths: A note. Macroeconomic Dynamics 17, 947-954.

Minsky, H.P., 1975. John Maynard Keynes. New York: Columbia University Press.

Minsky, H.P., 1982; 2016. Can It Happen Again? Essays on Instability and Finance. London: Routledge.

Minsky, H.P., 1986. Stabilising an Unstable Economy. New York: McGraw-Hill.

Nikolaidi, M., Stockhammer, E., 2017. Minsky models: A structured survey. Journal of Economic Surveys 31, 1304-1331.

Ninomiya, K., 2022. Financial structure, cycle, and instability. Journal of Economic Structures 11, 19.

Nishi, H., 2012. A dynamic analysis of debt-led and debt-burdened growth regimes with Minskian financial structure. Metroeconomica 63, 634-660.

Nishi, H., 2015. Comparative evaluation of post-Keynesian interest rate rules, income distribution and firms' debts for macroeconomic performance. Cambridge Journal of Economics 39, 189-219.

Nishi, H., 2019. An empirical contribution to Minsky's financial fragility: evidence from the non-financial sectors in Japan. Cambridge journal of economics. Economics 43, 585-622.

Nishi, H., Okuma, K., 2023. Fiscal policy and social infrastructure provision under alternative growth and distribution regimes. Evolutionary and Institutional Economics Review 20, 259-286.

Nishi, H., Okuma, K., 2024. A Kaleckian Growth Model with Public Capital and Debt Accumulation. Mmeo.

Nishi, H., Stockhammer, E., 2020. Distribution shocks in a Kaleckian model with hysteresis and monetary policy. Economic Modelling 90, 465-479.

Ono, T., 2020. Fiscal rules in a monetary economy: implications for growth and welfare. Journal of Public Economic Theory 22, 190-219.

Panizza, U., Presbitero, A.F., 2013. Public debt and economic growth in advanced economies: a survey. Swiss Journal of Economics and Statistics 149, 175-204.

Pedrosa, Í., Brochier, L., Freitas, F., 2023. Debt hierarchy: autonomous demand composition, growth and indebtedness in a supermultiplier model. Economic Modelling 126, 106369.

Reinhart, C.M., Rogoff, K.S., 2010. Growth in a time of debt. American Economic Review 100, 573-578.

Ryoo, S., 2010. Long waves and short cycles in a model of endogenous financial fragility. Journal of Economic Behavior and Organization 74, 163-186.

Sardoni, C., 2024. Public spending and growth: A simple model. Structural Change and Economic Dynamics, 69, 56-62.

Skott, P., 2023. Structuralist and Behavioral Macroeconomics. Cambridge University Press.

Sordi, S., Vercelli, A., 2006. Financial fragility and economic fluctuations. Journal of Economic Behavior & Organization, 61, 543-561.

Tavani, D., Zamparelli, L., 2017. Government spending composition, aggregate demand, growth, and distribution. Review of Keynesian Economics 5, 239-258.

Taylor, L., Proano, C.R., de Carvalho, L., Barbosa, N., 2012. Fiscal deficits, economic growth and government debt in the USA. Cambridge Journal of Economics 36, 189-204.

Woo, J., Kumar, M.S., 2015. Public debt and growth. Economica 82, 705-739.

Jong-Il, Y., Dutt, A.K., 1996. Government debt, income distribution and growth. Cambridge Journal of Economics 20, 335-351.

**Appendix 1 Proof of Proposition 1**

This dynamic system consists of Equations (17), (21), and (25), the steady-state evaluated Jacobian $\mathcal{J}^*$ elements of which are expressed as follows:

$$\mathcal{J}^* = \begin{pmatrix} -g^* & -\chi^*\left(1-\dfrac{\partial g_s}{\partial \chi}\right) & 0 \\ 0 & -\kappa^*\left(1-\dfrac{\partial g_d}{\partial g}\right) & 0 \\ -\sigma\phi\dfrac{g^*\hat{q}^*}{\chi^*} & \phi\hat{q}^*\left(1-\sigma+\sigma\dfrac{\partial g_s}{\partial g}\right) & -\phi\hat{q}^* \end{pmatrix}$$

The characteristic Equation for this Jacobian can be defined as follows:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{A1}$$

Where $\lambda$ denotes the characteristic root. The coefficients $a_1$ and $a_2$ and $a_3$ are given as follows:

$$a_1 = g^* + \phi\hat{q}^* + \kappa\left(1-\frac{\partial g_d}{\partial g}\right) \tag{A2}$$

$$a_2 = \kappa(g^* + \phi\hat{q}^*)\left(1-\frac{\partial g_d}{\partial g}\right) + \phi g^*\hat{q}^* \tag{A3}$$

$$a_3 = \kappa\phi g^*\hat{q}^*\left(1-\frac{\partial g_d}{\partial g}\right) \tag{A4}$$

where

$$1-\frac{\partial g_d}{\partial g} = \frac{sm(1-\tau)-(1+\delta^*)\gamma}{sm(1-\tau)} > 0 \tag{A5}$$

is guaranteed by Keynesian stability conditions. Note that the steady-state values are independent of $\kappa$ and $\phi$.

According to the Ruth-Hurwitz criterion, the necessary and sufficient conditions for steady-state local asymptotic stability are

$$a_1 > 0,\ a_2 > 0,\ a_3 > 0, \text{ and } a_1a_2 - a_3 > 0$$

First, we assume that $a_3$ is positive if $g^* > 0$ and $\hat{q}^* > 0$ are satisfied. When either of these conditions is not satisfied, the steady-state becomes a saddle point. Moreover, from the Domar stability condition, the steady state growth rate must be $g^* > i$, where $g^* = \frac{sm(\alpha+\beta m)(1-\tau)-\gamma i\delta^*}{sm(1-\tau)-(1+\delta^*)\gamma}$ and $\hat{q}^* > 0$ are equivalent to $g^* > n$. Thus, fiscal policy promotes economic growth and facilitates the fulfilment of these conditions.

If these conditions are satisfied and $a_3$ is positive, the values of $a_1$ and $a_2$ are positive from Equations (A2) and (A3), respectively. With further arrangements, it can be shown that

$$a_1a_2 - a_3 = (g^* + \phi\hat{q}^*)\left(g^* + \kappa\left(1-\frac{\partial g_d}{\partial g}\right)\right)\left(\phi\hat{q}^* + \kappa\left(1-\frac{\partial g_d}{\partial g}\right)\right) \tag{A6}$$

This is a quadratic function with respect to $\phi$ or $\kappa$. When $g^* > 0$ and $\hat{q}^* > 0$ are satisfied, then $\phi$ and $\kappa$ have a negative x-axis and a positive y-axis, respectively. Therefore, for $\phi > 0$ and $\kappa > 0$, $a_1 a_2 - a_3 > 0$ always holds.

Hence, three Keynesian stability conditions, the Domar stability condition, and a positive labour productivity growth rate are necessary for local asymptotic stability of the steady state. These are given by the following Equations, respectively.

$$sm(1-\tau) - (1+\delta^*)\gamma > 0 \tag{A7}$$

$$g^* > i \tag{A8}$$

$$g^* > n \tag{A9}$$

Q.E.D.

**Appendix 2 Determinants of economic growth rate**

The economic growth rate in the steady state is differentiated and organised according to each parameter. By considering the stable domain for interest rate $i_{\mathrm{K}}$ and the target debt ratio $\delta_{\mathrm{K}}$, as well as the positive capacity utilisation condition (48), we obtain the following results

$$\frac{dg^*}{d\tau} = \frac{sm\gamma\big((\alpha+\beta m)(1+\delta^*) - i\delta^*\big)}{(sm(1-\tau) - (1+\delta^*)\gamma)^2} > 0 \tag{$A10$}$$

$$\frac{dg^*}{d\delta^*} = \gamma \frac{sm(\alpha+\beta m)(1-\tau) - i(sm(1-\tau) - \gamma)}{(sm(1-\tau) - (1+\delta^*)\gamma)^2} > 0 \tag{$A11$}$$

$$\frac{dg^*}{di} = -\frac{\gamma\delta^*}{sm(1-\tau) - (1+\delta^*)\gamma} < 0 \tag{$A12$}$$

$$\frac{dg^*}{ds} = -\frac{(1-\tau)m\gamma\big((\alpha+\beta m)(1+\delta^*) - i\delta^*\big)}{(sm(1-\tau) - (1+\delta^*)\gamma)^2} < 0 \tag{$A13$}$$

$$\frac{dg^*}{dm} = \frac{G(m)}{(sm(1-\tau) - (1+\delta^*)\gamma)^2} \tag{$A14$}$$

where

$$G(m) \equiv a_g m^2 + b_g m + c_g \tag{$A15$}$$

in which $a_g \equiv s\beta(1-\tau)$, $b_g \equiv -2\beta\gamma(1+\delta^*)$, and $c_g \equiv -\alpha\gamma(1+\delta^*) + i\gamma\delta^*$. $G(m)$ is a quadratic function that is convex downwards in the $(m, G(m))$-plane, being valid in $m \in (\tilde{m}, 1)$, where $\tilde{m} \equiv \frac{(1+\delta^*)\gamma}{s(1-\tau)}$. Let $m_g$ be the positive profit share satisfying $G(m_g) = 0$. An economy has a wage-led growth regime in $m < m_g$ and a profit-led growth regime in $m > m_g$.

**Appendix 3 Impacts of saving rate, tax rate and profit share on the fragility boundaries**

Within the range of Keynesian stable target debt ratio, the impacts of saving rate $s$ and tax rate $\tau$ on the $i_{hs}(\delta^*)$-curve as follows:

$$\frac{di_{hs}(\delta^*)}{ds} = \frac{m(\alpha+\beta m)(1-\tau)(1+2\delta)}{\delta^*(sm(1-\tau)+\tau-\gamma-2\gamma\delta)^2}(\gamma\delta^*-\tau)<0 \tag{A16}$$

$$\frac{di_{hs}(\delta^*)}{d\tau} = \frac{(\alpha+\beta m)(1+2\delta^*)}{\delta^*(sm(1-\tau)+\tau-\gamma-2\gamma\delta^*)^2}\big((sm-\gamma)-\gamma(1+sm)\delta^*\big)>0 \tag{A17}$$

In this range, these impacts on the $i_{sp}(\delta^*)$-curve as follows:

$$\frac{di_{sp}(\delta^*)}{ds} = -\frac{m(\alpha+\beta m)(1+\delta^*)(1-\tau)\tau}{\delta^*\big(sm(1-\tau)+\tau-\gamma(1+\delta^*)\big)^2}<0 \tag{A18}$$

$$\frac{di_{sp}(\delta^*)}{d\tau} = \frac{(\alpha+\beta m)(1+\delta^*)}{\delta^*\big(sm(1-\tau)+\tau-\gamma(1+\delta^*)\big)^2}(sm-(1+\delta^*)\gamma)>0 \tag{A19}$$

Thus, a lower saving rate and a higher tax rate shifts $i_{hs}(\delta^*)$- and $i_{sp}(\delta^*)$-curves upwards. The impact of the profit share on these curves is unclear.

By differentiating the maximum interest rate satisfying the Domar stability condition $i_{\mathrm{K}}$ with respect to the saving rate, tax rate, and profit share, we obtain

$$\frac{\partial i_{\mathrm{K}}}{\partial s} = -\frac{m(\alpha+m\beta)\gamma(1-\tau)}{(sm(1-\tau)-\gamma)^2}<0 \tag{A20}$$

$$\frac{\partial i_{\mathrm{K}}}{\partial \tau} = \frac{sm(\alpha+m\beta)\gamma}{(sm(1-\tau)-\gamma)^2}>0 \tag{A21}$$

$$\frac{\partial i_{\mathrm{K}}}{\partial m} = \frac{s(1-\tau)\big(a_g m^2+b_{g0}m+c_{g0}\big)}{(sm(1-\tau)-\gamma)^2} \gtreqless 0 \tag{A22}$$

where $b_{g0}$ and $c_{g0}$ are the value of $b_g$ and $c_g$ at $\delta^*=0$. Comparing this with Equation (A15) shows that in a profit-led growth regime, an increase in the profit share always allows for a higher maximum interest rate, $i_{\mathrm{K}}$, whereas in a wage-led growth regime, the effect is ambiguous.

Similarly, by differentiating the maximum target debt ratio satisfying the Keynesian stability condition $\delta_{\mathrm{K}}$ with respect to these parameters, we obtain

$$\frac{\partial \delta_{\mathrm{K}}}{\partial s} = \frac{m(1-\tau)}{\gamma}>0 \tag{A23}$$

$$\frac{\partial \delta_{\mathrm{K}}}{\partial \tau} = \frac{-sm}{\gamma}<0 \tag{A24}$$

$$\frac{\partial \delta_{\mathrm{K}}}{\partial m} = \frac{s(1-\tau)}{\gamma}>0 \tag{A25}$$

Thus, the maximum target debt ratio $\delta_{\mathrm{K}}$ is increased by a rise in the saving rate and the profit share, as well as by a fall in the tax rate.

**Appendix 4 Three cases for fiscal fragility**

Appendix 4 identifies the conditions for three cases of fiscal fragility.

In Case 1, $\tilde{\delta}_{hs} < \tilde{\delta}_{sp} < \delta_{\mathrm{K}}$ is satisfied, which is equivalent to the condition that the slope of both $i_{hs}(\delta^*)$- and $i_{sp}(\delta^*)$- curves are upward-sloping at $\delta^* = \delta_{\mathrm{K}}$. In this case, a steady state exists for the hedge, speculative, and Ponzi positions.

In Case 2, $\tilde{\delta}_{hs} < \delta_{\mathrm{K}} < \tilde{\delta}_{sp}$ is satisfied, which is equivalent to the condition that the slope of $i_{sp}(\delta^*)$-curve is downward-sloping whereas that of $i_{hs}(\delta^*)$-curve is still upward-sloping at $\delta^* = \delta_{\mathrm{K}}$. In this case, a steady state exists for hedging and speculation, but the Ponzi position is always unstable.

In Case 3, $\tilde{\delta}_{hs} < \tilde{\delta}_{sp} < \delta_{\mathrm{K}}$ is satisfied, which is equivalent to the condition that the slope of both $i_{hs}(\delta^*)$- and $i_{sp}(\delta^*)$- curves are downward-sloping at $\delta^* = \delta_{\mathrm{K}}$ In this case, the stable steady state for hedge exists, but speculative and Ponzi positions are always unstable.

Which of these cases occurs can be identified by investigating the slope for $i_{hs}(\delta^*)$- and $i_{sp}(\delta^*)$- curves at $\delta^* = \delta_{\mathrm{K}}$. These are

$$\left.\frac{di_{hs}(\delta^*)}{d\delta^*}\right|_{\delta_{\mathrm{K}}} = \frac{(\alpha+\beta m)\gamma}{(sm(1-\tau)-\gamma)^2(sm(1-\tau)-\gamma-\tau)} H_K(x) \tag{A26}$$

$$\left.\frac{di_{sp}(\delta^*)}{d\delta^*}\right|_{\delta_{\mathrm{K}}} = \frac{(\alpha+\beta m)\gamma}{(sm(1-\tau)-\gamma)^2\tau} S_K(x) \tag{A27}$$

respectively, and $m$, $s$, and $\tau$ can be substituted in placeholders $x$ in $H_K(x)$ and $S_K(x)$.

**A4.1 Impacts of the income distribution $m$**

First, we consider the impact of income distribution $m$. Considering $H_K(x)$ and $S_K(x)$ as functions of $m$, we have

$$H_K(m) = 2s^2(1-\tau)^2m^2 - 2s\gamma(1-\tau)m - \gamma\tau \tag{A28}$$

$$S_K(m) = s^2(1-\tau)^2m^2 - s\gamma(1-\tau)m - \gamma\tau \tag{A29}$$

The graphs of $H_K(m)$ and $S_K(m)$ are parabolically convex downwards. The solution for $H_K(m) = 0$, which takes a positive value, is.

$$\tilde{m}_{hs}^{+} = \frac{s\gamma(1-\tau)+\sqrt{s^2\gamma(1-\tau)^2(\gamma+2\tau)}}{2s^2(1-\tau)^2} \tag{A30}$$

Therefore, the slope of $i_{hs}(\delta^*)$ is downwards in $m < \tilde{m}_{hs}^{+}$, whereas it is upwards in $m > \tilde{m}_{hs}^{+}$.

The solutions to $S_K(m) = 0$, which takes a positive value is

$$\tilde{m}_{sp}^{+} = \frac{\gamma + \sqrt{\gamma(\gamma + 4\tau)}}{2s(1-\tau)} \tag{A31}$$

Therefore, the slope of $i_{sp}(\delta^*)$ is downwards in $m < \tilde{m}_{sp}^{+}$, whereas it is upwards in $m > \tilde{m}_{sp}$.

To clarify the positional relationship between $H_K(m)$ and $S_K(m)$, we verify

$$S_K(\tilde{m}_{hs}^{+}) = -\frac{\gamma\tau}{2} \tag{A32}$$

and

$$H_K\left(\tilde{m}_{sp}^{+}\right) = \gamma\tau \tag{A33}$$

Thus, $\tilde{m}_{hs} < \tilde{m}_{sp}^{+}$ always holds true. Furthermore, because both the target debt ratio and the profit share must be positive while satisfying the Keynesian stability condition, it follows that

$$m > m_K \equiv \frac{\gamma}{s(1-\tau)} \tag{A34}$$

Naturally, $m_K < \tilde{m}_{hs}^{+} < \tilde{m}_{sp}^{+}$ is established.

Hence, *ceteris paribus*, the following three cases can be established at $\delta_{\mathrm{K}}$. First, when $m < \tilde{m}_{hs}^{+}$ the slope of the $i_{hs}(\delta^*)$-curve and $i_{sp}(\delta^*)$-curve is both downwards, and accordingly, there exist steady states in which only the hedge fiscal position is stable, but the speculative and Ponzi fiscal positions are always unstable. Second, when $\tilde{m}_{hs}^{+} < m < \tilde{m}_{sp}^{+}$ the slope of the $i_{hs}(\delta^*)$-curve is upwards, while the slope of the $i_{sp}(\delta^*)$-curve is downwards; stable steady states exist for hedge and speculative fiscal positions, but the Ponzi fiscal position is always unstable. Third, when $\tilde{m}_{sp}^{+} < m$ the slopes of the $i_{hs}(\delta^*)$-curve and $i_{sp}(\delta^*)$-curve is both upwards, and accordingly, there exist steady states in which all hedge, speculative, and Ponzi fiscal positions are stable.

### A4.2 Impacts of Saving rate $s$

As $H_K(m)$ and $S_K(m)$ can be regarded as functions of $s$ given other variables, we defined them as $H_K(s)$ and $S_K(s)$, respectively. The solution to $H_K(s) = 0$, which takes a positive value, is

$$\tilde{s}_{hs}^{+} = \frac{m\gamma(1-\tau) + \sqrt{m^2\gamma(1-\tau)^2(\gamma + 2\tau)}}{2m^2(1-\tau)^2} \tag{A35}$$

It follows that the slope of $i_{hs}(\delta^*)$ is downwards in $s < \tilde{s}_{hs}^{+}$ and it is upwards in $s > \tilde{s}_{hs}^{+}$.

Similarly, the solutions to $S_K(s) = 0$, which takes a positive value, is

$$\tilde{s}_{sp}^{+} = \frac{\gamma + \sqrt{\gamma(\gamma + 4\tau)}}{2m(1-\tau)} \tag{A36}$$

Therefore, the slope of $i_{sp}(\delta^*)$ is downwards in $s < \tilde{s}_{sp}^{+}$ and it is upwards in $s > \tilde{s}_{sp}^{+}$.

For these, to clarify the positional relationship between $H_K(s)$ and $S_K(s)$, we verify

$$S_K(\tilde{s}_{hs}^{+}) = -\frac{\gamma\tau}{2} \tag{A37}$$

and

$$H_K(\tilde{s}_{sp}^{+}) = \gamma\tau \tag{A38}$$

As the graphs of $H_K(s)$ and $S_K(s)$ are parabolically convex downwards, $\tilde{s}_{hs}^{+} < \tilde{s}_{sp}^{+}$ is always true. Furthermore, because both the target debt ratio and saving rate must be positive while satisfying the Keynesian stability condition, it follows that

$$s > s_K \equiv \frac{\gamma}{m(1-\tau)} \tag{A39}$$

Naturally, $s_K < \tilde{s}_{hs}^{+} < \tilde{s}_{sp}^{+}$ is established.

Hence, *ceteris paribus*, the following three cases can be established at $\delta_K$. First, when $s < \tilde{s}_{hs}^{+}$ the slopes of the $i_{hs}(\delta^*)$-curve and $i_{sp}(\delta^*)$-curve are downward, there exist steady states in which only the hedge fiscal position is stable, but the speculative and Ponzi fiscal positions are always unstable. Second, when $\tilde{s}_{hs}^{+} < s < \tilde{s}_{sp}^{+}$ the slope of the $i_{hs}(\delta^*)$-curve is upwards, while the slope of the $i_{sp}(\delta^*)$-curve is downward; stable steady states exist for hedge and speculative fiscal positions, but the Ponzi fiscal position is always unstable. Third, when $\tilde{s}_{sp}^{+} < s$ the slopes of the $i_{hs}(\delta^*)$-curve and $i_{sp}(\delta^*)$-curve is both upwards, and accordingly, steady states exist in which all of the hedge, speculative, and Ponzi fiscal positions are stable.

### A4.3 Impacts of tax rate $\tau$

Let $H_K(x)$ and $S_K(x)$ be as a function of tax rate $\tau$. Then, their graphs of $H_K(\tau)$ and $S_K(\tau)$ are parabolas convex downwards. Both solutions to $H_K(\tau) = 0$ takes the following positive values

$$\tilde{\tau}_{hs}^{++} = \frac{2sm(2sm-\gamma) + \gamma + \sqrt{\gamma}\sqrt{\gamma + 4sm(sm(2+\gamma)-\gamma)}}{4s^2m^2} \tag{A40}$$

$$\tilde{\tau}_{hs}^{+} = \frac{2sm(2sm-\gamma) + \gamma - \sqrt{\gamma}\sqrt{\gamma + 4sm(sm(2+\gamma)-\gamma)}}{4s^2m^2} \tag{A41}$$

At the maximum tax rate, we have $H_K(1) = -\gamma < 0$. Therefore, the larger value $\tilde{\tau}_{hs}^{++}$ is greater than unity. It follows that the slope of $i_h(\delta)$ is upwards in $\tau < \tilde{\tau}_{hs}^{+}$, and it is downwards in $\tilde{\tau}_{hs}^{+} < \tau$.

The two solutions to $S_K(\tau) = 0$ take the following positive values,

$$\tilde{\tau}_{sp}^{++} = \frac{2s^2m^2 + (1-sm)\gamma + \sqrt{\gamma}\sqrt{\gamma + sm(sm(4+\gamma)-2\gamma)}}{2s^2m^2} \tag{A42}$$

$$\tilde{\tau}_{sp}^{+} = \frac{2s^2m^2 + (1-sm)\gamma - \sqrt{\gamma}\sqrt{\gamma + sm(sm(4+\gamma)-2\gamma)}}{2s^2m^2} \tag{A43}$$

At the maximum tax rate, we have $S_K(1) = -\gamma < 0$. Therefore, a larger value $\tilde{\tau}_{sp}^{++}$ is greater than unity. It follows that the slope of $i_{sp}(\delta)$ is upwards in $\tau < \tilde{\tau}_{sp}^{+}$, and it is downwards in $\tilde{\tau}_{sp}^{+} < \tau$.

For these, to clarify the positional relationship between $H_K(\tau)$ and $S_K(\tau)$, we verify

$$S_K(\tilde{\tau}_{hs}^{+}) = -\frac{\gamma\left(\gamma + 2sm(2sm - \gamma) - \sqrt{\gamma}\sqrt{\gamma + 4sm(sm(2+\gamma) - \gamma)}\right)}{8s^2m^2} < 0 \qquad (A44)$$

and

$$H_K(\tilde{\tau}_{sp}^{+}) = \frac{\gamma\left(2s^2m^2 + (1 - sm)\gamma - \sqrt{\gamma}\sqrt{\gamma + sm(sm(4+\gamma) - 2\gamma)}\right)}{2s^2m^2} > 0 \qquad (A45)$$

As the graphs of $H_K(\tau)$ and $S_K(\tau)$ are parabolas convex downwards, $\tilde{\tau}_{sp}^{+} < \tilde{\tau}_{hs}^{+}$ is always true. Furthermore, because both the target debt ratio and the tax rate must be positive while satisfying the Keynesian stability condition, it follows that

$$\tau < \tau_K \equiv \frac{sm - \gamma}{sm} \qquad (A46)$$

Naturally, $\tau_K < 1$ and $\tilde{\tau}_{sp}^{+} < \tilde{\tau}_{hs}^{+} < \tau_K$ are established.

Hence, *ceteris paribus*, the following three cases can be established at $\delta_{\mathrm{K}}$. First, when $\tau < \tilde{\tau}_{sp}^{+}$ the slopes of the $i_{hs}(\delta^*)$-curve and $i_{sp}(\delta^*)$-curve are upwards, there exist steady states in which all the hedge, speculative and Ponzi fiscal positions are stable. Second, when $\tilde{\tau}_{sp}^{+} < \tau < \tilde{\tau}_{sh}^{+}$ the slope of the $i_{hs}(\delta^*)$-curve is upwards, while the slope of $i_{sp}(\delta^*)$-curve is downwards, there exist stable steady states for hedge and speculative fiscal positions, but the Ponzi fiscal position is always unstable. Third, when $\tilde{\tau}_{hs}^{+} < \tau$ the slope of $i_{hs}(\delta^*)$-curve and $i_{sp}(\delta^*)$-curve are downwards, there exist steady states in which only the hedge fiscal position is stable, but the speculative and Ponzi fiscal positions are always unstable.