Macroprudential policy in a dynamic stochastic disequilibrium model

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Abstract

The main objective of this article is to examine the effects of macroprudential policy in the context of a theoretical model, in particular, a dynamic stochastic disequilibrium (DSDE) model. The DSDE model is a dynamic and stochastic general equilibrium model with intertemporal optimization of the agents in the economy, which is similar to dynamic stochastic general equilibrium (DSGE) models. The economic models are populated by households (active and inactive). The active households face the risk of dropping out of the labor force (i.e., risk permanent income loss), losing labor and profit sources of income and becoming inactive. Once the household is inactive, it cannot reenter the labor force and thus faces a probability of death. A firm sells final consumer goods to households in a perfectly competitive market and buys differentiated goods from a continuum of intermediate firms. These intermediate firms borrow from households to finance a part of their capital stock. The central bank sets monetary policy using a Taylor rule. The macroprudential policy is modeled in two ways: directly including a reserve requirement in the budget constraint of the active households or including a financial intermediary entity (a bank) in which the central bank retains a portion of bank funding. The model assumes that nominal wage inflation is subject to bargaining between workers and firms, that is, labor markets do not clear through an accommodating nominal wage. The DSDE model indicates that macroprudential policy also influences the system and is able to reduce credit, prices and output in the system.

Key-words: Disequilibrium model. Macroprudential. Reserve requirement.
JEL Classification: E17.E58.E61.

Introduction

The general objective of this article is to analyze an economy through dynamic models with microfoundations that exhibit intertemporal optimization and rational expectations. The construction of a DSGE model that incorporates the general and dynamic aspects and behavioral interactions of firms, consumers and governments allows for the analysis and simulation of the economy. Thus, the objective is to elaborate a model that facilitates the evaluation of the impacts of economic policies, including fiscal, monetary, and macroprudential policies[1], thereby allowing the analysis of fluctuations in the main macroeconomic variables.

The use of DSGE models for macroeconomic analysis is recent. Few studies have analyzed the mechanisms through which exogenous shocks, mainly in the financial sector, are transmitted to the real economy, and the relationship between these policies and reductions in financial instability remains obscure. Many DSGE models have already explored the monetary purpose of the instruments, although, in general, they focus only on the interest rate. Numerous questions remain about the implementation and regulation of these macroeconomic models in the financial sector.

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[1] The macroprudential policy debate incorporates instruments that are used periodically in the pursuit of the financial stability of the overall economic system (GALATI; MOESSNER, 2011).

The present article intends to extend the DSDE model of Schoder (2016) to include a simple macroprudential policy hypothesis adapted from Vinhado and Divino (2016) and to simulate the model responses by calibrating the parameters. The DSDE model is similar to a DSGE model in that its microfoundations feature intertemporal optimization and rational expectations. The DSDE model assumes that wage rate inflation is fixed by a collective bargaining process between firms and workers' representatives and that the risk of income loss faced by the household is permanent. Thus, the household accumulates precautionary savings. The former assumption implies that the nominal wage is not an equilibrating variable as in the DSGE model, and the latter assumption indicates a Keynesian consumption function relating consumption to current income and wealth. Finally, we add a monetary authority adopting an reserve requirement (RR) to control the supply of funds directly to the household as a macroprudential policy problem in the first model. In the second, we add a financial intermediary that lends to firms using a supply of funds from the household. The central bank retains a percentage of the bank's supply of funds as a non remunerated reserve requirement. The remainder of this article is organized as follows. Section 1 provides some references for the DSGE model. Section 2 describes the DSDE model. Section 3 presents the results: first explaining the implications of the DSDE model, second indicating the model parametrization, and finally, displaying the simulation exercises for the DSDE model. Section 4 introduces DSDE model with a financial intermediary and model simulation. Conclusions are provided in the final section of this article.

1 Theoretical references

Many DSGE models in New Keynesian economics seek to introduce rigidity and shocks such that economic theory approaches reality and, consequently, fits the observed data. Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003) are references for DSGE models following the adjustment principle, which pursue adherence to the observed data (COSTA-JUNIOR, 2015). Models developed at IPEA (VEREDA; CAVALCANTI, 2010; CAVALCANTI; VEREDA, 2011) and the BCB (CASTRO et al., 2011), called SAMBA, consider large and medium-sized firms and are reference models compatible with the Brazilian economic context.

Some articles initially explored the assumptions that characterize the financial sector and are included in DSGE models. For example, the financial accelerator hypothesis (BERNANKE; GERTLER; GILCHRIST, 1999) and more complex forms, such as the banking sector (BRZOZA-BRZEZINA; MAKARSKI, 2011) and banking conglomerates (GERALI et al., 2010), consider the inclusion of financial friction on the demand and supply sides of credit.

Regarding financial sector models, Bernanke, Gertler and Gilchrist (1999) present one of the first DSGE models to include financial friction. The authors also include the financial accelerator mechanism such that the focus of financial friction is on the demand side of credit and the financial system amplifies the economic cycle. Brzoza-Brzezina and Makarski (2011) develop a model with more financial sector elements, including friction in the demand for credit requiring collateral from the borrower and a banking sector that charges different funding and lending rates than the established policy rate.

A second aspect of the literature explores another element of economic policy and regulation: stabilizing the financial system as a whole through macroprudential policy. That is, some papers study the role of macroprudential policies and attempt to incorporate them into the modeling. Some economists (WOODFORD, 2011; SVENSSON, 2012) discuss the need to analyze the impacts of macroeconomic policies through modeling that includes a macroeconomic framework that is separate from monetary policy, with its own instruments and objectives (policy goals) to reduce the systemic risk of the overall economy. Some of the instruments that can be incorporated are compulsory deposits, CRs, government-imposed taxes, and capital controls in the case of open economies.

Moreover, many recent articles highlight the role of macroprudential policy, its instruments and its objectives, including Galati and Moessner (2011), Vinals et al. (2011), Collin et al. (2014), ENGLAND (2009), Borio (2011), Shin (2010), CGFS (2010), among others. These authors agree that the general objective of macroprudential policy is financial stability. Galati and Moessner (2011) provide one definition of financial stability: a financial system that is robust to external shocks. This means a financial system that is resilient to normal-sized shocks such that it can recover and perform its standard functions (SILVA; SALES; GAGLIANONE, 2013; BORIO; DREHMANN, 2009; VINALS et al., 2011).

To evaluate the conduct of macroeconomic policy, several papers analyze the role of macroprudential instruments in the economy and in the financial system as a whole. Some show that many countries use these as auxiliary instruments of monetary policy and/or as tools of financial stability. Brzoza-Brzezina, Kolasa and Makarski (2013) introduce CRs to examine their impact on the economic system using a DSGE model for the Eurozone; moreover, Ferreira (2016), Brandi (2013) do so for the Brazilian system, Glocker and Towbin (2012), Agénor, Silva and Awazu (2011) assess the impact of the RR using a DSGE model in the Brazilian context, and Vinhado and Divino (2016) include the two instruments cited above in their model.

2 Model

The DSDE model proposed by Schoder (2016) is the reference for our analysis. The DSDE is a DSGE model with modifications to the labor market and household problems. In the latter case, the household faces an uninsurable risk of permanent income loss that creates precautionary saving motives, which in turn lead to a consumption function that is related to current income and wealth. In the former case, the nominal wage is not an equilibrium variable that clears the labor market. The financial system is included in the model based on the adaptations of Vinhado and Divino (2016). The central bank sets monetary policy and macroprudential rules to ensure financial stability.

The economic model is populated by households (economically active and inactive). The final good firm sells consumer goods to households in a perfectly competitive market and buys differentiated goods from a continuum of intermediate firms. These intermediate firms borrow a part of the capital stock from the households. The central bank sets monetary and macroprudential policy, aiming first for a Taylor rule with inflation stabilization. For macroprudential policy, the central bank directly regulates the economically active households' wealth by setting an RR in the budget constraint. The model also considers the government budget to be balanced at all times. The model assumes that the labor market does not clear through an accommodating nominal wage; in particular, nominal wage inflation is subject to a bargaining process between workers and firms. In the simplest case, the rate of wage inflation is constant. The economy grows at a deterministic rate given by labor-embodied productivity Γ. (Notation: $\tilde{X}_t \equiv \frac{X_t}{\Gamma^t}$ for any aggregated variable X_t.)

2.1 Households

There are two types of households: active and inactive. The first are born into generations of constant size (size 1) and are part of the labor force. They face a per-period risk U of permanent income loss, that is, the risk of dropping out of the labor force, losing labor and profit sources of income and becoming economically inactive. Once a household is inactive, it cannot return to the labor force. An inactive household faces a per-period probability of death D. [2]

The representative inactive household i does not work or obtain income and faces a per-period probability of death (independent of age). The source of income is the wealth the household accumulated during the active state. The stock of wealth is equal across households entering the

[2] The law of motion of the active population size $\Theta_{a,t}$ is $\Theta_{a,t} - \Theta_{a,t-1} = 1 - U\Theta_{a,t-1}$, and that of the inactive population size $\Theta_{i,t}$ is $\Theta_{i,t} - \Theta_{a,t-1} = U\Theta_{a,t-1} - D\Theta_{it-1}$. The steady-state values are $\Theta_a = 1/U$ and $\Theta_i = 1/D$.

inactive state at the same time $(t-s)$, but it differs between households that become inactive at different times. Here, $b_{i,t-s,t}$ is the end-of-period-t real wealth held by an inactive household when it becomes inactive in $t-s$ with $s \in \{0, 1, ..., \infty\}$. The household sells wealth at the beginning of period t ($R_t P_t b_{i,t-s,t}$, where R_t and P_t are the nominal interest rate and the price level, respectively) to an insurance market in a framework similar to that proposed by Blanchard (1985) and considering that the household may die between t and $t+1$ (that is, at beginning of period $t+1$). Thus, the inactive household receives a flow value of bequests from the insurance company (of households still alive $R_{t-1}P_{t-1}b_{i,t-s,t-1}$ and of dead households, whose wealth is distributed by the insurance company to households that are still alive $DR_{t-1}P_{t-1}b_{i,t-s,t-1}$). The problem of inactive households is to choose the optimal paths for consumption ($c_{i,t-s,t}$) and wealth ($b_{i,t-s,t}$) subject to the budget constraint[3] and conditional on staying alive in a dynamic program of the following form:

$$V_i(b_{i,t-s,t-1}) = \max_{c_i,t-s,t} \{lnc_{i,t-s,t-1} + \beta(1-D)E_t V_i(b_{i,t-s,t})\}$$
$$s.t. \quad c_{i,t-s,t} + b_{i,t-s,t} = \frac{R_{t-1}(1+D)}{\Pi_{p,t}} b_{i,t-s,t-1}$$

where β is the discount factor, $\Pi_{p,t}$ is the gross rate of price inflation, $V_i(b_{i,t-s,t-1})$ is the value function in t, $b_{i,t-s,t-1}$ is the state variable, and E_t is the expectations operator. The model derivation is provided in Annex A1. The model aggregation is available in Schoder (2016).

The following equations display the problem of the active households as a dynamic program. The problem is to choose optimal paths for consumption (c_a, t) and wealth $(b_{a,t})$ subject to the budget constraint. Active households consume and face a risk U of dropping out of the labor force (i.e., the risk of permanent income loss), and to insure against this risk, the active households will accumulate precautionary savings.[4] They have the same wealth regardless of when they were born, and they cannot die and lose their income in the same period. The labor supply will be fixed at n, and every active household is affected by unemployment:

$$V_a(b_{a,t-1}) = \max_{c_a,t} \{lnc_{a,t-1} + \beta(1-U)E_t V_a(b_{a,t}) + \beta U E_t V_i(b_{a,t})\}$$
$$s.t. \quad c_{a,t} + b_{a,t} = \omega(1-u_t)n + \pi_{d,t} - t_t - \tau_t + \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1} - \mu_t b_{a,t}$$

where ω_t, u_t, $\pi_{d,t}$ and t_t are the real wage, the unemployment rate, distributed profits, and a lump-sum government tax, respectively; $V_a(b_{a,t-1})$ is the value function in t; $b_{a,t-1}$ is the state variable; $V_i(b_{a,t})$ is the value function of active households that become inactive at the beginning of t+1; and τ_t is a simplification for aggregation. It is assumed that wealth is equally distributed across active non-newborn and newborn households at any point in time. Therefore, a newborn household receives $b_{a,t} - \tau_t$; in other words, the non-newborn transfer is τ_t of household wealth. [5] Macroprudential policy is represented as $\mu b_{a,t}$. The central bank charges μ of the household's wealth as a direct RR to control the supply of funds.

Calculating the first-order conditions (FOCs) of the household problem and their aggregation yields the following main equations. The FOCs of an inactive household are as in equation 1, which

[3] Budget constraint: $P_t c_{i,t-s,t} + P_t b_{i,t-s,t} = R_{t-1}P_{t-1}b_{i,t-s,t-1} + DR_{t-1}P_{t-1}b_{i,t-s,t-1}$.

[4] The adoption of the savings buffer setup is from Carroll (1997), Rabitsch and Schoder (2016) in which the household faces the risk of permanent income loss and will optimally accumulate precautionary savings. One of the main findings is that the level of steady-state consumption in the buffer stock savings model is lower than in conventional models.

[5] Schoder (2016) assumes that the transfer is financed by a tax on wealth ($\tau_t = \tau b_{a,t}$), and hence, $\tau = U$. This is because $(1/U - 1)\tau b_{a,t} = b_{a,t} - \tau b_{a,t}$; that is, the payment aggregated over all non-newborn active households must equal the receipts aggregated over all newborn active households.

proportionately relates their consumption in period t to their beginning-of-period wealth by a factor κ, where $\kappa = (1 - \beta(1 - D))$. Since an active household internalizes the inactive household's solution, this gives rise to a crucial property of the household problem: a Keynesian-type consumption function at the steady state. This is because the consumption choice of an inactive household is proportional to its previously accumulated real wealth:

$$\tilde{C}_{i,t} = \kappa \frac{1}{\Gamma} \frac{R_{t-1}}{\Pi_{p,t}} \left(\tilde{B}_{i,t-1} + U \tilde{B}_{a,t-1} \right) \tag{1}$$

where $\tilde{C}_{i,t}$, $\tilde{B}_{i,t}$ and $\tilde{B}_{a,t}$ are the aggregate consumption of inactive households, the real wealth of inactive households and the real wealth of active households, respectively (all de-trended). Note that $U\tilde{B}_{a,t-1}$ is the wealth that became inactive at the beginning of t. Equation 27 is the aggregate budget constraint of the inactive household, and 3 is the aggregate budget constraint of the active household, which are normalized by the trend Γ^t. Only active households receive wage and profit income, whereas newly inactive households receive funds from wealth carried over as a previously active household:

$$\tilde{C}_{i,t} + \tilde{B}_{i,t} = \frac{1}{\Gamma} \frac{R_{t-1}}{\Pi_{p,t}} \left(\tilde{B}_{i,t-1} + U \tilde{B}_{a,t-1} \right) \tag{2}$$

$$\tilde{C}_{a,t} + (1 + \mu_t) \tilde{B}_{a,t} = \tilde{Z}_t + (1 - U) \frac{1}{\Gamma} \frac{R_{t-1}}{\Pi_{p,t}} \tilde{B}_{a,t-1} \tag{3}$$

where the active household's de-trended aggregate real net income is

$$\tilde{Z}_t = \tilde{\omega}_t L_t + \tilde{\Pi}_{d,t} - \tilde{T}_t \tag{4}$$

where $\tilde{\Pi}_{d,t}$ is the distributed profits of intermediate good firms, and $\tilde{T}_t$ is the lump-sum tax. The labor input L_t is derived from the definition of the unemployment rate:

$$1 - u_t = \frac{L_t}{N} \tag{5}$$

Finally, the FOC w.r.t. consumption yields the aggregate consumption Euler equation for the active households in which the discounted expected marginal utility of consumption in the next period includes the risk of permanent income loss and depends on real wealth. This equation is implied by the substitution of the inactive household FOCs for the active household FOCs, observing that the term $\frac{R_t}{\Pi_{p,t+1}} \kappa \tilde{B}_{a,t}$ is the consumption of the newly inactive household in t+1[6]:

$$\frac{1}{\tilde{C}_{a,t}} = \beta(1 - U) \frac{1}{\Gamma} E_t \frac{R_t}{\Pi_{p,t+1}} \frac{1}{1 + \mu_t} \frac{1}{\tilde{C}_{a,t+1}} + \beta U \frac{1}{\kappa \tilde{B}_{a,t}} \tag{6}$$

This equation can be written as follows:

$$\frac{1}{\tilde{C}_{a,t}} = \beta E_t \frac{1}{\Gamma} \frac{R_t}{\Pi_{p,t+1}} \left(\frac{(1 - U)}{(1 + \mu_t)} \frac{1}{\tilde{C}_{a,t+1}} + U \frac{1}{\frac{R_t}{\Pi_{p,t+1}} \kappa \tilde{B}_{a,t}} \right) \tag{7}$$

Final aggregate consumption is defined by

$$\tilde{C}_t = \tilde{C}_{a,t} + \tilde{C}_{i,t} \tag{8}$$

[6] Appendix A1 shows the household model derivation.

2.2 Firms

The representative final good firm buys a wide variety of intermediate inputs and sell them as a unique basket of consumption goods to households in a perfectly competitive market. The final good firm uses good y_t as an input and takes the price p_t as given for a continuum of differentiated intermediate firms. The cost minimization problem of the final good firms leads to a demand for the intermediate good given the final good technology, the price of the intermediate good and the demand for the final good. In other words, the cost minimization problem is subject to the final good firm's production function and leads to an inverse relationship between the input price and the demand for a given output of final goods:

$$\min_{\{y_t\}} \int_0^1 p_t y_t dy$$
$$s.t. \quad Y_t = \int_0^1 \left(y_t^{\frac{\epsilon-1}{\epsilon}} dy \right)^{\frac{\epsilon}{\epsilon-1}}$$

where Y_t is final good output, and ϵ is the elasticity of substitution of inputs (given by technology and final good demand).[7] The demand for the intermediate good can be obtained by the FOC w.r.t. $y_{i,t}$:

$$y_t = \left(\frac{p_t}{P_t}\right)^{-\epsilon} Y_t$$

The intermediate good demand depends on the price of the intermediate good relative to the price of the final good.

There is an infinite continuum of intermediate good firms. The representative firm sells differentiated goods to the final firm with some market power under monopolistic competition; it is thus a price maker. The intermediate good y_t is produced using a constant-returns-to-scale Cobb-Douglas technology combining labor l_t and capital k_{t-1} as inputs and facing quadratic adjustment costs in investment and prices. The production function is the following:

$$y_t = (\Gamma k_{t-1})^{\alpha} (\Gamma^t l_t)^{1-\alpha}$$

where $0 \leq \alpha \leq 1$ is the output elasticity of capital, and $\Gamma^t l_t$ is the the labor-embodied productivity that grows at a deterministic rate. At the end of period t, the intermediate firm produces y_t using labor l_t and capital k_{t-1} as inputs (given capital adjustment costs) and chooses the capital good k_t to be used in production in the subsequent period. The model includes a firm-specific capital assumption in which firms own their capital stock, and their initial capital stock cannot be reduced. For instance, the firm's capital cannot be rented or sold in the market because it is specific to a given firm. This is the case, for example, among real estate companies and firms that use specialized equipment. The following capital accumulation equation states that the capital stock available at the end of period t is equal to the capital stock available in the last period, k_{t-1}, net of period-t capital stock depreciation, δK_{t-1}, with δ being the depreciation rate, plus the amount of capital accumulated during period t, which is determined by investments made during that period i_t:

$$k_t = i_t + (1 - \delta) k_{t-1}$$

In addition, the intermediate firm's problem includes how much of its investment will be financed externally, that is, the supply of bonds d_t. Firms obtain resources from households, issuing

[7] The price elasticity of substitution of inputs is the relative change in the ratio of the intermediate goods $y_{i,t}$ over the relative change in the ratio of the according price $y_{i,t}$. The cost minimization problem is subject to the Dixit-Stiglitz(1977) aggregator, which aggregates technology including monopolistic competition and increasing returns in the short run. The elasticity is invariant to the quantity of intermediate firms (infinite number), and thus, the elasticity and markup are constant.

bonds equal to the number of units of capital k_t, where the price of one unit of capital is equal to q_t. This constraint depends on a fixed portion of capital goods, measured as an efficient price:

$$d_t = \lambda q_t k_t$$

where λ is the target debt-capital ratio, and q_t is Tobin's q.

Furthermore, the intermediate firm faces adjustment costs in prices and capital. We assume quadratic costs for both costs. This means that the adjustment costs increase disproportionally faster than the amount of capital (or good prices) to be adjusted. Additionally, the function assumes zero value when there is no shock to increase the cost and when the investment covers depreciation (in the case of price adjustments, when prices do not change). In other words, in setting the nominal prices of their goods and installing new capital in their factories, firms face costly adjustments. The capital and price adjustment costs are assumed to evolve according to the respective equations:

$$\frac{\tau_i}{2}\left(\frac{i_t}{\Gamma k_{t-1}} - (1-(1-\delta)\frac{1}{\Gamma})\right)^2 k_{t-1} \quad and \quad \frac{\tau_p}{2}\Gamma^t\left(\frac{p_t}{p_{t-1}} - \Pi\right)^2$$

where τ_i and τ_p are the adjustment cost scaling parameters, namely, the degree of capital and price rigidity, respectively.

The optimization problem is that the intermediate firm chooses intertemporal paths for prices, labor demand, investment, capital stock and the supply of bonds ($\{p_t, l_t, i_t, k_t, d_t\}_{t=0}^{\infty}$, respectively) to maximize the discounted sum of expected future distributed profits while taking the total output, the overall price level, the predetermined capital stock and the laws of motion of capital, nominal wages, the production function, the demand function for intermediate goods and the target debt-capital ratio as given:

$$\max_{\{p_t,l_t,i_t,k_t,d_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{P_0}{P_t}\Lambda_{0,t} \quad \left\{ \begin{array}{c} p_t y_t - \omega_t l_t - P_t i_t - P_t \frac{\tau_i}{2}\left(\frac{i_t}{\Gamma k_{t-1}} - (1-(1-\delta)\frac{1}{\Gamma})\right)^2 k_{t-1} - \\ -P_t \frac{\tau_p}{2}\Gamma^t\left(\frac{p_t}{p_{t-1}} - \Pi\right)^2 + P_t d_t - R_{t-1}P_{t-1}d_{t-1} \end{array} \right\}$$

$$\begin{aligned} s.t. \quad & k_t = i_t + (1-\delta)k_{t-1} \\ & y_t = (\Gamma k_{t-1})^{\alpha}(\Gamma^t l_t)^{1-\alpha} \\ & y_t = \left(\frac{p_t}{P_t}\right)^{-\epsilon} Y_t \\ & d_t = \lambda q_t k_t \end{aligned}$$

where ω is the nominal wage per unit of labor, and Λ is the stochastic discount factor (the value of a unit of real profit at time t+j in terms of the value of a unit of real profit at time t). The FOC w.r.t. d_t is

$$\begin{aligned} \frac{P_t}{P_t}\Lambda_{t,t}P_t + \frac{P_t}{P_t}\Lambda_{t,t}P_t\eta - E_t\frac{P_t}{P_{t+1}}\Lambda_{t,t+1}R_t P_t = 0 \\ 1 + \eta - E_t\frac{R_t}{\Pi_{p,t+1}}\Lambda_{t,t+1} = 0 \\ \eta = 0 \end{aligned} \quad (9)$$

where

$$E_t\Lambda_{t,t+1} = \beta E_t \frac{(1-U)c_{a,t+1}^{-1} + U(1+D)c_{i,t+1,t+1}^{-1}}{c_{a,t}^{-1}} = E_t\left(\frac{R_t}{\Pi_{p,t+1}}\right)^{-1}$$

where η is the Lagrange multiplier with respect to d_t. The firm finances part of its capital acquisition in each period from the resources of the households. Changes in the firm's financing structure, for example, in the amount of cash borrowed, are not relevant from the household perspective. For instance, the household will not charge a higher interest rate for a larger amount of borrowed capital.

The FOC w.r.t. p_t describes the evolution of price setting. In addition, intermediate firms set the same price $p_t = P_t$; thus, a continuum of a mass one of firms produces $y_t = Y_t$:

$$
\begin{aligned}
(1-\epsilon)y_t - P_T\tau_p\Gamma^t\left(\frac{p_t}{p_{t-1}} - \Pi\right)\frac{1}{p_{t-1}} + \epsilon P_t\varphi_t\frac{y_t}{p_t} + \\
+E_t\frac{P_t}{P_{t+1}}\Lambda_{t,t+1}P_{t+1}\tau_p\Gamma^{t+1}\left(\frac{p_{t+1}}{p_t} - \Pi\right)\frac{p_{t+1}}{p_t^2} = 0 \\
(1-\epsilon)Y_t - \tau_p\Gamma^t(\Pi_t - \Pi)\Pi_t + \epsilon\varphi_t Y_t + E_t\Lambda_{t,t+1}\tau_p\Gamma^{t+1}(\Pi_{p,t+1} - \Pi)\Pi_{p,t+1} = 0 \\
((\epsilon-1) - \epsilon\varphi_t)\tilde{Y}_t + \tau_p(\Pi_t - \Pi)\Pi_t - E_t\Lambda_{t,t+1}\tau_p\Gamma(\Pi_{p,t+1} - \Pi)\Pi_{p,t+1} = 0
\end{aligned} \tag{10}
$$

All intermediate firms set their prices using the same markup over the same marginal costs. The elasticity of substitution of inputs is derived from the final firm demand for intermediate goods and determines the markup that the intermediate firms charge over their marginal cost (given the price adjustment cost). Consequently, the intermediate firm has a certain degree of market power in monopolistic competition and can set the prices of goods. Nevertheless, price setting incorporates nominal adjustment costs, as noted by Rotemberg (1982). In other words, the model includes nominal price rigidity in which prices are adjusted more slowly than is ideal. This adjustment cost leads to good prices that are lower than they would be without nominal rigidity, and as a result, the markup over marginal costs is also smaller. The first term in equation 9 refers to the logic of the markup over the marginal cost; the last two terms are related to the price adjustment cost.

The FOCs w.r.t. i_t and k_t show that the intermediate firm's decision is related to investment in terms of the capital stock by the shadow price of investment q_t, namely, by Tobin's theory of investment. It expresses the capital price measured as capital profitability and comes from the constraint – the law of motion of capital – from the firm's optimization problem (Lagrangian multiplier). For instance, the firm will optimally choose i_t given by q_t. In other words, the optimal investment choice in t depends on the value of q_t. Tobin's q in this model implies the marginal value of an additional unit of capital in terms of profits, taking into account capital adjustment costs; that is, when increasing investment today by one unit, equation 10 (the marginal loss of an increase in i_t) must be equal to equation 11 (the marginal gain due to an extra unit of capital in the subsequent period). In other words, q_t "measure[s] how much profits the firm would gain by having one more unit of capital installed in the next period" Schoder (2016).

The FOC w.r.t. i_t is as follows:

$$
\begin{aligned}
-P_t - P_t\tau_i\left(\frac{i_t}{\Gamma k_{t-1}} - \left(1 - (1-\delta)\frac{1}{\Gamma}\right)\right)\frac{1}{\Gamma k_{t-1}}k_{t-1} + P_t q_t = 0 \\
q_t = 1 + \tau_i\frac{1}{\Gamma}\left(\frac{i_t}{\Gamma k_{t-1}} - \left(1 - (1-\delta)\frac{1}{\Gamma}\right)\right) \\
q_t = 1 + \tau_i\frac{1}{\Gamma}\left(\frac{\tilde{I}_t}{\tilde{K}_{t-1}} - \left(1 - (1-\delta)\frac{1}{\Gamma}\right)\right)
\end{aligned} \tag{11}
$$

The FOC w.r.t. k_t is as follows:

$$P_t q_t + P_t \eta_{t+1} \lambda q_t = E_t \frac{P_t}{P_{t+1}} \Lambda_{t,t+1} \begin{bmatrix} P_{t+1}\tau_i \left(\frac{i_{t+1}}{\Gamma k_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)\frac{i_{t+1}}{\Gamma k_t} - \\ -P_{t+1}\frac{\tau_i}{2}\left(\frac{i_{t+1}}{\Gamma k_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)^2 + \\ P_{t+1}\varphi_{t+1}\alpha(\Gamma k_t)^{\alpha-1}(\Gamma^t l_{t+1})^{1-\alpha} + \\ +P_{t+1} q_{t+1}(1-\delta) \end{bmatrix}$$

$$q_t = E_t \Lambda_{t,t+1} \begin{bmatrix} \tau_i \left(\frac{i_{t+1}}{\Gamma k_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)\frac{i_{t+1}}{\Gamma k_t} - \\ -\frac{\tau_i}{2}\left(\frac{i_{t+1}}{\Gamma k_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)^2 + \\ +\varphi_{t+1}\alpha\left(\frac{\Gamma k_t}{\Gamma^t l_{t+1}}\right)^{\alpha-1} + q_{t+1}(1-\delta) \end{bmatrix}$$

$$q_t = E_t \Lambda_{t,t+1} \begin{bmatrix} \tau_i \left(\frac{\tilde{I}_{t+1}}{\tilde{K}_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)\frac{\tilde{I}_{t+1}}{\tilde{K}_t} - \\ -\frac{\tau_i}{2}\left(\frac{\tilde{I}_{t+1}}{\tilde{K}_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)^2 + \\ +\varphi_{t+1}\alpha\left(\frac{\tilde{Y}_{t+1}}{\Gamma\tilde{K}_t}\right)^{\alpha-1} + q_{t+1}(1-\delta) \end{bmatrix} \tag{12}$$

Equations 11 and 12 refer to the logic of q_t. Equation 12 reveals that the long-run desired level of capital is dependent on the long-run level of output (expected sales) $\tilde{Y}_{t+1}$, real wages (represented as real marginal costs φ_{t+1}) and real interest rates (represented in $E_t\Lambda_{t,t+1}$). The short-run dynamics of the desired capital stock are implied by the equations given changes in those variables. For instance, an increase in the real wage given a certain level of output (sales) and a real interest rate leads to an increase in the desired capital stock (substitution effect). Alternatively, an increase in the real interest rate reduces the desired capital stock given the real wage and the level of output. Note that the short-run dynamics of the desired capital stock, that is, the response of investment to shocks and how long a response takes, also depend on the adjustment costs.

The FOC is obtained w.r.t. the labor demand l_t. The production function can be rewritten as follows: $\left(\frac{yt}{\Gamma k_{t-1}}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\Gamma k_{t-1}}{\Gamma^t l_{t-1}}\right)^{-\alpha}$. The real marginal cost can be obtained:

$$-\omega_t + P_t\varphi_t\Gamma^{t(1-\alpha)}(1-\alpha)l_t^{-\alpha}(\Gamma k_{t-1})^{\alpha} = 0$$

$$\varphi_t = \frac{\omega_t}{P_t}\frac{1}{\Gamma^t}\frac{1}{1-\alpha}\left(\frac{\Gamma k_{t-1}}{\Gamma^t l_t}\right)^{-\alpha}$$

$$\varphi_t = \frac{\omega_t}{P_t}\frac{1}{\Gamma^t}\frac{1}{1-\alpha}\left(\frac{y_t}{\Gamma k_{t-1}}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\varphi_t = \tilde{\omega}_t\frac{1}{1-\alpha}\left(\frac{\tilde{Y}_t}{\tilde{K}_{t-1}}\right)^{\frac{\alpha}{1-\alpha}} \tag{13}$$

The de-trended real distributed profits are aggregated as follows. Observing that λ is the debt-capital ratio maintained by the intermediate good firms yields

$$\tilde{\Pi}_{d,t} = \tilde{Y}_t + \tilde{\omega}_t L_t + (1-\lambda)\tilde{I}_t + \\ +\frac{\tau_i}{2}\frac{1}{\Gamma}\left(\frac{\tilde{I}_t}{\tilde{K_{t-1}}} - (1-(1-\delta)\frac{1}{\Gamma})\right)^2\tilde{K}_{t-1} - \frac{\tau_p}{2}\left(\Pi_{p,t}-\Pi\right)^2 \tag{14}$$

2.3 Macroprudential policy

The model includes an RR as a macroprudential instrument in the form of a fraction μ_t of active household wealth $b_{a,t}$ that is held by the central bank. The macroprudential policy implemented by the central bank obeys the following rule in which μ reacts positively to variations in credit volume d_t:

$$\frac{\mu_t}{\mu} = \left(\frac{D_t}{D_{t-1}}\right)^{\varphi_{d,\mu}} V_{\mu,t} \tag{15}$$

where $V_{\mu,t}$ is an exogenous macroprudential policy shock.

2.4 Monetary and fiscal policy

The DSDE model includes government expenditures following an autoregressive process with an assumption of a budget that is balanced at all times and financed by lump-sum taxes:

$$\tilde{T}_t = \tilde{G}_t \tag{16}$$

$$\frac{\tilde{G}_t}{\tilde{G}} = V_{G,t} \tag{17}$$

where $\tilde{G}$ is steady-state government spending, and $V_{G,t}$ is an exogenous fiscal policy shock. The monetary authority defines monetary and macroprudential rules to ensure monetary and financial stability, respectively. The monetary authority sets the interest rate according to a simple Taylor rule that considers only inflation stabilization:

$$\frac{R_t}{R} = \left(\frac{\Pi_{p,t}}{\Pi}\right)^{\phi_{r\pi}} V_{R,t} \tag{18}$$

where R is the steady-state policy rate, Π is the steady-state inflation rate, $\phi_{r\pi}$ measures the elasticity (sensitivity) of the interest rate to deviations in inflation from the steady-state rate, and $V_{R,t}$ is a monetary policy shock.

2.5 Market clearing and model closure

The DSGE model assumes an accommodating nominal wage that ensures that labor supply equals labor demand. The DSDE has a Keynesian closure in the sense that the rate of wage inflation is subject to a bargaining process between worker and firm representatives, with a Nash solution to the optimization problem. In other words, nominal wage inflation is a non-accommodating variable in the labor market:

$$\max_{\Pi_{\omega,t}} (\tilde{\omega}(\Pi_{\omega,t}))^{\nu_t} (r(\Pi_{\omega,t}))^{1-\nu_t}$$

with

$$\frac{\nu_t}{\nu} = (1-u_t)^{\phi_u} V_{\nu,t} \tag{19}$$

where V_t is an exogenous shock to bargaining power, $\tilde{\omega}(\Pi_{\omega,t})$ is the steady-state real wage, and $r(\Pi_{\omega,t})$ is steady-state profit rate. These last two variables represent the return functions of workers and firms, respectively, where the former increases and the latter decreases with the rate of wage inflation. Thus, this is a game between the return functions of two agents concerned with the long-run implications of the bargaining process. The FOC shows the evolution of the desired rate of wage inflation ($\Pi_{\omega,t}$):

$$1 = (1 - 1/\nu_t) \frac{\tilde{\omega}(\Pi_{\omega,t})}{r(\Pi_{\omega,t})} \frac{r'(\Pi_{\omega,t})}{\tilde{\omega}'(\Pi_{\omega,t})} \tag{20}$$

The parameter ϕ_u can assume values of 0 or 2; that is, it is equal to 2 when there is feedback from the labor market to wage formation and 0 when there is no feedback effect. Thus, the rate of wage inflation is constant. The relationship between the growth rate of real wage and wage or price inflation is given by

$$\frac{\tilde{\omega}_t}{\tilde{\omega}_{t-1}} - 1 = \Pi_{w,t} - \Pi_{p,t} \tag{21}$$

The macroeconomic balance condition that aggregate demand equals output ensures that the good market clears:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t + \frac{\tau_i}{2}\frac{1}{\Gamma}\left(\frac{\tilde{I}_t}{\tilde{K}_{t-1}} - (1-(1-\delta)\frac{1}{\Gamma})\right)^2 \tilde{K}_{t-1} - \frac{\tau_p}{2}\left(\Pi_{p,t} - \Pi\right)^2 \tag{22}$$

Therefore, this model hypothesizes that the labor market is not cleared by the nominal wage, and the consequence is output determination by aggregate demand.

2.6 Exogenous shocks

The following equations exhibit the assumption of autoregressive exogenous shocks to monetary policy, fiscal policy, worker bargaining power and macroprudential policy. They evolve as follows:

$$V_{R,t} = (V_{R,t-1})^{\rho_R} \exp \varepsilon_{R,t} \tag{23}$$
$$V_{G,t} = (V_{G,t-1})^{\rho_G} \exp \varepsilon_{G,t} \tag{24}$$
$$V_{\mu,t} = (V_{\mu,t-1})^{\rho_\mu} \exp \varepsilon_{\mu,t} \tag{25}$$
$$V_{\nu,t} = (V_{\nu,t-1})^{\rho_\nu} \exp \varepsilon_{\nu,t} \tag{26}$$

where $\varepsilon_{R,t}$, $\varepsilon_{G,t}$, $\varepsilon_{\mu,t}$ and $\varepsilon_{\nu,t}$ are assumed to be exogenous, i.i.d. and normal innovations ($\sim N(0,1)$).

3 Model results

The representation of the economy as a general equilibrium model with linearized equations derived from the dynamic optimization problem of the agents in the economy is subject to some restrictions, is not a trivial methodology and is dependent on a numerical solution and a state-space representation. Therefore, the adoption of these models requires the researcher to use complex mathematical methods to solve the optimization problems of the agents, including new hypotheses and linear approximations of these conditions around a chosen point, the steady state. Non-trivial effort is exerted to obtain the numerical solution to the model and its simulation to investigate the dynamics of the relationships of interest. This process essentially depends on the calibration of the parameters and the assignment of values compatible with economic reality. The following section describes the parameter calibration and out-of-steady-state dynamics to analyze the economy characterized by the DSDE model, as well as the model implications of the DSDE hypothesis.

3.1 Implications of the hypothesis and steady state of the DSDE model

The intertemporal consumption problem includes the risk to active households of dropping out of the labor force. Moreover, active households consider the solution to the inactive household problem (consumption proportional to real wealth). In the Euler equation, the active household's expected utility of consumption depends on its previously accumulated wealth. The standard Euler equation includes consumption dependent on individual preferences; in contrast, the DSDE model includes current wealth as related to current consumption. Therefore, the DSDE Euler equation can be called a Keynesian-type consumption function (CARROLL, 1997; RABITSCH; SCHODER, 2016; SCHODER, 2016).[8] This can be seen since there is a equilibrium relationship among income, wealth and consumption. The equilibrium consumption and wealth conditional on income can be calculated:

> Then, we have two equations (the Euler equation and budget constraint of the active household) in the consumption-income ratio and the wealth-income ratio for which the existence of a unique solution can be shown under certain parameter constellations. As we can see, introducing the risk of permanent income loss to the conventional consumer problem implies the existence of an equilibrium consumption-income ratio and wealth-income ratio. With rising income, consumption will increase by a fixed proportion, which is very similar to a Keynesian consumption function. (SCHODER, 2016, p.6)

Another implication of the DSDE Euler equation is related to the interest rate. In the standard DSGE model with zero unemployment, the natural interest rate is implied by a parameter restriction on the Euler equation. The central bank's target interest rate also needs to equal the natural interest rate derived from the Euler equation for the steady state to be obtained. The DSDE model has a different logic. The steady-state interest rate (derived from the Euler equation) needs to equal the target rate; however, the former rate is not implied solely by parameter restrictions. There is a new element of the Euler equation in which consumption is related to wealth and the real interest rate, that is, the steady-state interest rate is implied by the steady states of other variables. In other words, the real interest rate is implied by almost the entire system. Is this the natural interest rate? Not always. In the steady state, the unemployment rate can be different from zero, and with the assumption of a constant nominal wage (without labor feedback), the labor market will not be cleared by the nominal wage, leading to a steady-state interest rate that is different from a steady state in which there is no unemployment. The steady-state interest rate is calculated by using a given wage inflation rate that may or may not equal the natural rate (labor market clearing).

The steady state of the model is calibrated with unemployment of 10%. This means that the steady-state interest rate is not equal to the natural rate in this calibration. It is important to note an additional implication of the interest rate mechanism. The real interest rate has to be lower than the deterministic growth rate for the existence of a meaningful steady state in the DSDE model. This restriction stems from the precautionary savings motive that needs a stock of wealth that grows slower than income; otherwise, wealth would increase exponentially. Schoder (2017b) alludes to the fact that the DSDE model is dynamically inefficient (a criterion of Pareto efficiency), as the resources are underutilized in equilibrium.

Moreover, the nominal wage is not an accommodating variable that adjusts to clear the labor market, i.e., there is unemployment that is not merely frictional, and nominal wage movements do not entirely eliminate unemployment. As a consequence, business fluctuations (output changes) are not given by changes in the factor markets. This means that the equilibrium adjustment of aggregate output occurs through aggregate spending, which is demand-driven adjustment. Although there is an effect of supply shocks on demand and output through labor market feedback (a supply shock changes relative bargaining power, nominal wage inflation and, consequently, demand), the output is determined by aggregate demand. This is a feature of disequilibrium models.

[8] "In comparison with the standard model, the buffer-stock model predicts a much higher marginal propensity to consume out of transitory income, a much higher effective discount rate for future labor income, and a positive rather than negative sign for the correlation between saving and expected labor income growth" Rabitsch and Schoder (2016, p.2).

3.2 Parameter calibration

The main parameters of the model are calibrated as in Schoder (2016). In the relevant part of the parameterization, it is appropriate to conduct a literature review of the Brazilian and international literature on the assigned values. The values can be based on the relevant literature and/or estimated via Bayesian techniques and the information contained in the data. This procedure is intended to test the ability of the model to fit the data and to represent stylized facts. The objective of this chapter is to first conduct a simulation exercise using the values assigned by Schoder while adding the parameter values related to macroprudential policy, particularly the RR, as in Vinhado and Divino (2016).

Table 2 lists the parameters and the values they were assigned for the model simulation. The probability of income loss for an active household (U) is such that the old-age dependency ratio[9] is 0.3, that is, $U/D = 0.3$, κ is implied by $(1-\beta(1-D))$, and N is the aggregate labor supply calibrated such that 10% of the labor force is unemployed in the steady state. Here, $\phi_{v,u}$ takes the value 0 for no labor market feedback and 2 for labor market feedback.

Regarding central bank macroprudential policy, ϕ_μ assumes the value 0.255, as stipulated in Vinhado and Divino (2016). The central bank's response to credit volume fluctuations is assumed to be tepid (1/4 of the increase/decrease in credit volume) because the RR is only somewhat frequently used. This parameter is the only value calibrated as in Vinhado and Divino (2016), and we have defined a steady-state RR of 20% (0.2). The remaining parameters are defined as in Schoder (2016).

Table 1 – Parameter calibration

Parameters	Definition	Value
Household		
Γ	Growth factor of labor-embodied productivity	1.01
β	Household discount rate	0.998
D	Probability of death (inactive households)	0.002
U	Probability of income loss (active households)	0.0006
κ	Steady-state consumption-wealth ratio of inactive households	0.004
N	Aggregate labor supply	0.65376
Firm		
α	Output elasticity of capital	0.4
δ	Rate of capital depreciation	0.025
ϵ	Elasticity of substitution of intermediate goods	3
λ	Target debt-capital ratio	0.15
τ_i	Investment adjustment cost scaling parameter	30
τ_p	Price adjustment cost scaling parameter	100
Central Bank		
$\phi_{r,\pi}$	Inflation elasticity of the interest rate	1.1
Π_p	Target inflation rate of the monetary authority	1
R	Interest target of the monetary authority	1.002
$\tilde{G}$	Steady-state government expenditures	1
ρ_G	Persistence of a government spending shock	0.7
μ	Steady-state reserve requirement	0.2
ϕ_μ	Response of the reserve requirement to credit oscillations	0.255
Bargaining problem		
$\phi_{\nu,u}$	Unemployment elasticity of the workers' bargaining power	0 and 2
ρ_ν	Persistence of wage inflation	0.7

Source: (SCHODER, 2016; VINHADO; DIVINO, 2016)

9 The number of elderly people as a share of those of working age.

3.3 Impulse-response analysis

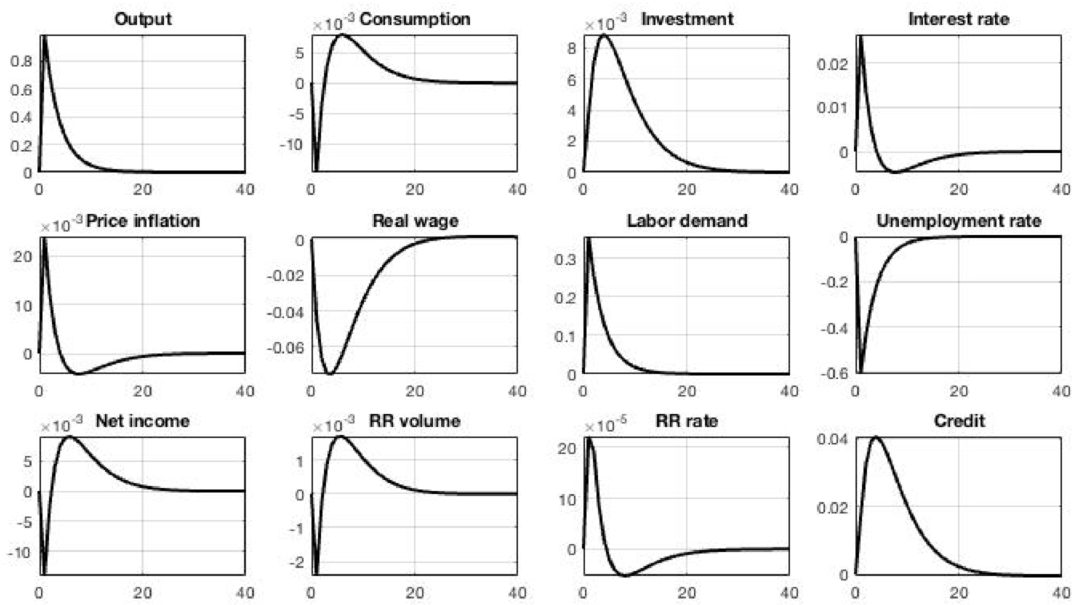
This section presents a model simulation exercise for macroeconomic variables in the DSDE model given as exogenous shocks. The predicted macroeconomic effect is revealed by an impulse-response analysis over an interval of forty quarters. The model dynamics are reported given an exogenous shock to budget-neutral fiscal policy, monetary policy, macroprudential policy and worker bargaining power. The main idea is to understand not only the effect of macroprudential policy but also the overall transmission mechanism of the model. The following figures display the responses to exogenous policy economic shocks that cause deviations from the steady state.

3.3.1 Fiscal policy shock

Figure 1 plots the impulse-response functions for a 1% budget-neutral government spending shock. The baseline model assumes constant wage inflation (unemployment elasticity of the workers' bargaining power $\phi_{\nu,u} = 0$) and lump-sum taxes of the same amount as government expending (the purpose is to maintain a balanced budget). The model dynamics work similarly to those in a textbook Keynesian model, with multiplier effects in output, consumption and investment for twenty quarters and without crowding-out effects for the last two variables. Output increases by approximately 1.00 units and consumption increases by approximately 0.08 units in the sixth quarter, and investment increases by approximately 0.09 units in the fourth quarter. Consumption and net income initially decrease after an interest rate hike.

Despite higher taxes and RRs at the beginning of the period, increases in disposable income allow households to spend more on consumption goods. The increases in credit volume and higher prices reduce the real wage at a given nominal wage, which in turn decreases the unemployment rate and stimulates investment. The interest rate increases, reducing prices and elevating real wages. The propagation mechanism is the adjustment not only of prices but also of quantities.

Figure 1 – Responses to a 1% persistent budget-neutral government spending shock

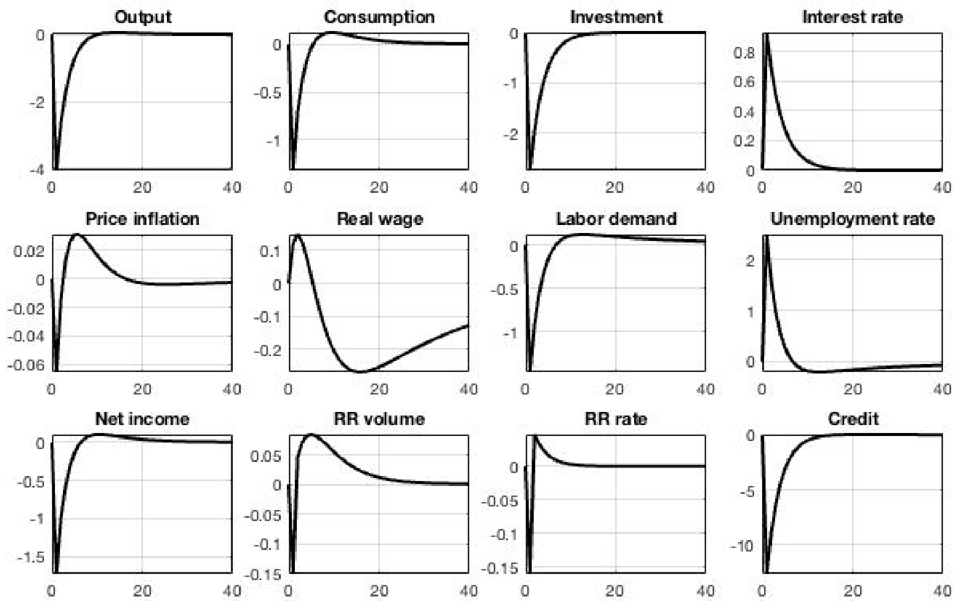


Source: Author's own elaboration.

3.3.2 Monetary policy shock

The transmission mechanism for a 1% interest rate shock in the economic model is displayed in Figure 2. Output, consumption and investment decrease, and the interest rate increases moderately as prices return to their steady-state values. Disposable income and credit volume decrease, as does the RR volume. The real wage increases given the initial fall in prices, but since firms consider real marginal costs when setting the prices of intermediate goods, the prices eventually increase and the real wage decreases, with some persistence after twenty periods. The prices and capital adjustment costs also contribute persistence to the aggregate demand variables and their components as they return to their steady-state levels. An increase in real wages is sufficient only for returning investment to the steady-state level. The substitution effect does not work in the initial period because, given predetermined capital, a higher real wage is correlated with a decrease in the output-capital ratio (Yt/Kt-1 - equation 13), which in turn reduces the initial output. A decrease in investment is associated with a rising real interest rate that reduces the desired capital stock.

Figure 2 – Responses to a 1% persistent interest rate shock



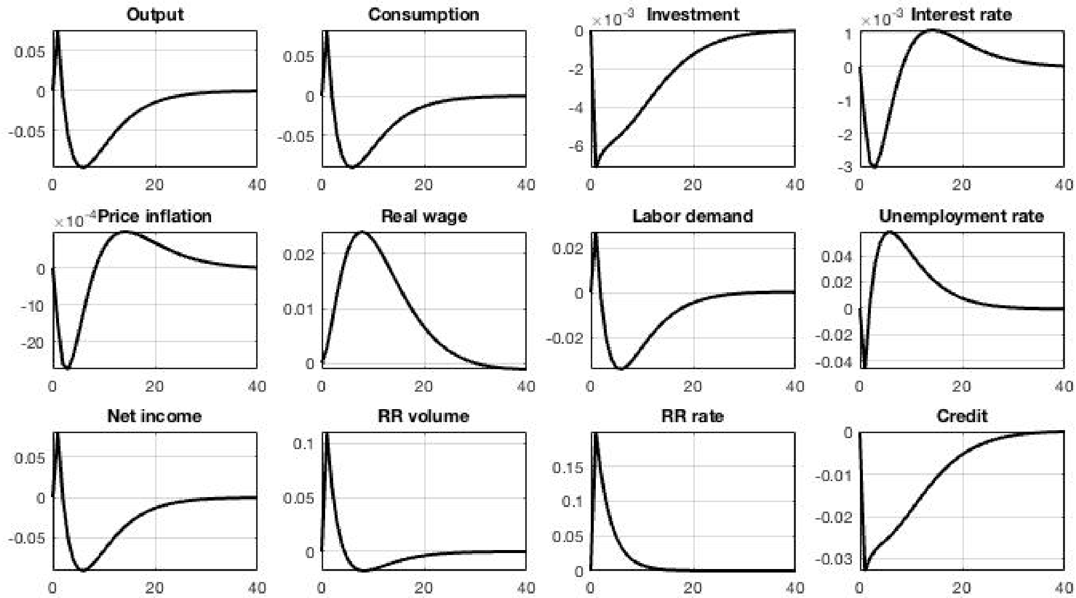
Source: Author's own elaboration.

3.3.3 Macroprudential policy shock

The effects of an exogenous contractionary RR shock in the DSDE model are depicted in Figure 3. The macroprudential policy shock has the opposite effects on macroeconomics variables as a government expenditure shock. A higher RR acts similarly to a lump-sum tax in the model, decreasing disposable income and, given multiplier effects, reducing output, investment and consumption. The credit volume also decreases. With an initial decrease in prices, the consequence is an increase in real wages and an increase in the unemployment rate. Returning to equilibrium after a macroprudential policy shock takes approximately as long as recovery following a fiscal policy shock.

Vinhado and Divino (2016) note that macroprudential policy could be either a substitute for or a complement to monetary policy given its effects on lower inflation in their model. The decrease in prices following a contractionary RR rate shock is reproduced in the DSDE model, which is consistent with the related literature on the effects of the economic cycle.

Figure 3 – Responses to a 1% persistent reserve requirement shock

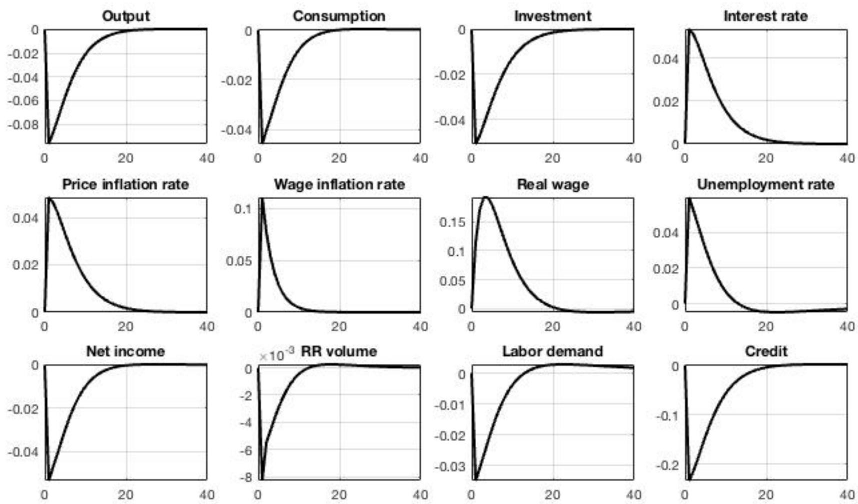


Source: Author's own elaboration.

3.3.4 Worker bargaining power shock

Figure 4 shows the impulse-response functions for a 1% persistent shock to worker bargaining power in the model. The increase in price inflation induces an increase in real wages and a subsequent decrease in aggregate demand and its components. Given the labor supply and capital stock, increases in real wages and in the real interest rate are sufficient to increase real marginal costs and reduce the expected profits of firms (evaluated as Tobin's q), causing increases in unemployment and price inflation and decreases in income, credit volume and RR volume.

Figure 4 – Responses to a 1% persistent worker bargaining power shock



Source: Author's own elaboration.

4 Bank model

The DSDE model is modified to include a financial intermediary that lends to intermediate firms using financial resources from active households. The household problem is identical to that in in Schoder (2016), that is, the FOCs are the almost identical to those in the section 2.1; the difference is that we exclude the term $\mu_t b_{a,t}$ from the active households' budget constraint. The supply of funds of active households ($b_{a,t}$) serves as bank funding. The aggregate main equations of the household problem are as follows:

Budget constraint

$$\tilde{C}_{i,t} + \tilde{B}_{i,t} = \frac{1}{\Gamma}\frac{R_{t-1}}{\Pi_{p,t}}\left(\tilde{B}_{i,t-1} + U\tilde{B}_{a,t-1}\right) \tag{27}$$

$$\tilde{C}_{a,t} + \tilde{B}_{a,t} = \tilde{Z}_t + (1-U)\frac{1}{\Gamma}\frac{R_{t-1}}{\Pi_{p,t}}\tilde{B}_{a,t-1} \tag{28}$$

An active household's de-trended aggregate real net income is

$$\tilde{Z}_t = \tilde{\omega}_t L_t + \tilde{\Pi}_{d,t} - \tilde{T}_t \tag{29}$$

The labor input L_t is

$$1 - u_t = \frac{L_t}{N} \tag{30}$$

Aggregate consumption Euler equation

$$\frac{1}{\tilde{C}_{a,t}} = \beta(1-U)\frac{1}{\Gamma}E_t\frac{R_t}{\Pi_{p,t+1}}\frac{1}{\tilde{C}_{a,t+1}} + \beta U\frac{1}{\kappa\tilde{B}_{a,t}} \tag{31}$$

Final aggregate consumption

$$\tilde{C}_t = \tilde{C}_{a,t} + \tilde{C}_{i,t} \tag{32}$$

4.1 Intermediate firm problem

The optimization problem of intermediate firms has some modifications. The bank will charge a different rate of interest on d_t (R_t^L). The optimization problem reads as follows:

$$\max_{\{p_t,l_t,i_t,k_t,d_t\}_{t=0}^{\infty}} E_0\sum_{t=0}^{\infty}\frac{P_0}{P_t}\Lambda_{0,t} \quad \left\{\begin{array}{c} p_t y_t - \omega_t l_t - P_t i_t - P_t\frac{\tau_i}{2}\left(\frac{i_t}{\Gamma k_{t-1}} - (1-(1-\delta)\frac{1}{\Gamma})\right)^2 k_{t-1} - \\ -P_t\frac{\tau_p}{2}\Gamma^t\left(\frac{p_t}{p_{t-1}} - \Pi\right)^2 + P_t d_t - R_{t-1}^L P_{t-1} d_{t-1} \end{array}\right\}$$

$$\begin{aligned} s.t. \quad & k_t = i_t + (1-\delta)k_{t-1} \\ & y_t = (\Gamma k_{t-1})^{\alpha}(\Gamma^t l_t)^{1-\alpha} \\ & y_t = \left(\frac{p_t}{P_t}\right)^{-\epsilon} Y_t \\ & d_t = \lambda q_t k_t \end{aligned}$$

Then, the FOC w.r.t d_t does not imply $\mu_t = 0$. Instead, μ represents a relationship between the two rates of interest:

$$\frac{P_t}{P_t}\Lambda_{t,t}P_t + \frac{P_t}{P_t}\Lambda_{t,t}P_t\eta - E_t\frac{P_t}{P_{t+1}}\Lambda_{t,t+1}R_t^L P_t = 0$$

$$1 + \eta - E_t\frac{R_t^L}{\Pi_{p,t+1}}\Lambda_{t,t+1} = 0$$

$$\eta = \frac{R_t^L}{R_t} - 1 \tag{33}$$

The firm's financing structure is relevant for the bank optimization problem when including a second rate of interest. Moreover, the FOC w.r.t k_t includes this new element (η) different from zero, that is, q_t also considers the relationship between the two interest rates:

$$P_t q_t + P_t \eta_{t+1} \lambda q_t = E_t \frac{P_t}{P_{t+1}} \Lambda_{t,t+1} \begin{bmatrix} P_{t+1}\tau_i \left(\frac{i_{t+1}}{\Gamma k_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)\frac{i_{t+1}}{\Gamma k_t} - \\ -P_{t+1}\frac{\tau_i}{2}\left(\frac{i_{t+1}}{\Gamma k_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)^2 + \\ P_{t+1}\varphi_{t+1}\alpha(\Gamma k_t)^{\alpha-1}(\Gamma^t l_{t+1})^{1-\alpha} + \\ +P_{t+1}q_{t+1}(1-\delta) \end{bmatrix}$$

$$q_t = E_t \Lambda_{t,t+1} \begin{bmatrix} \tau_i \left(\frac{i_{t+1}}{\Gamma k_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)\frac{i_{t+1}}{\Gamma k_t} - \\ -\frac{\tau_i}{2}\left(\frac{i_{t+1}}{\Gamma k_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)^2 + \\ +\varphi_{t+1}\alpha\left(\frac{\Gamma k_t}{\Gamma^t l_{t+1}}\right)^{\alpha-1} + q_{t+1}(1-\delta) \end{bmatrix} * 1/(1+\eta_{t+1}\lambda)$$

$$q_t = E_t \Lambda_{t,t+1} \begin{bmatrix} \tau_i \left(\frac{\tilde{I}_{t+1}}{\tilde{K}_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)\frac{\tilde{I}_{t+1}}{\tilde{K}_t} - \\ -\frac{\tau_i}{2}\left(\frac{\tilde{I}_{t+1}}{\tilde{K}_t} - \left(1-(1-\delta)\frac{1}{\Gamma}\right)\right)^2 + \\ +\varphi_{t+1}\alpha\left(\frac{\tilde{Y}_{t+1}}{\Gamma \tilde{K}_t}\right)^{\alpha-1} + q_{t+1}(1-\delta) \end{bmatrix} * 1/(1+\eta_{t+1}\lambda) \tag{34}$$

4.2 Commercial Banks

The commercial bank problem is adapted from Vinhado and Divino (2016) model. The comercial banks raise funds from active household wealth $B_{a,t}$. Loans are granted to financial firms D_t to raise funds for capital production. A part of bank funding is retained by the central bank as a non-remunerated reserve requirement $RR_t = \mu_t B_{a,t}$, where $\mu_t \in (0,1)$. The accounting identity of the bank balance sheet and the aggregate amount of loans (D_t) are, respectively,

$$RR_t + D_t = B_{a,t} + \Pi_t^b \tag{35}$$

$$D_t = \lambda q_t K_t \tag{36}$$

Where Π_t^b is the bank profit/loss and can be represented by the following equation:

$$\Pi_t^b = \frac{1}{\Gamma^2}\frac{R_{t-1}^L}{\Pi_{p,t}} D_{t-1} - \frac{1}{\Gamma^2}\frac{R_{t-1}}{\Pi_{p,t}} B_{a,t-1} \tag{37}$$

The first element in the second term is the amount of loans granted including financial revenues less resources captured from active households and including financial expenses. The commercial bank maximizes an objective function, in which the optimization problem is to choose the supply and demand of funds to maximize profits Π_t^b subject to the bank's accounting identity equation 36 and aggregate loan amount equation 36:

$$\max_{\{D_t, B_{a,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{P_0}{P_t}\Lambda_{0,t}\left[\frac{1}{\Gamma^3}P_{t-1}R_{t-1}^L D_{t-1} - \frac{1}{\Gamma^3}P_{t-1}R_{t-1}B_{a,t-1}\right]$$

The FOC w.r.t D_t implies

$$R_t^L = \frac{R_t}{(1-\mu)} \tag{38}$$

The solution is an equation representing the interest rate for loan operations in association with the rate that pays interest on active household wealth plus a spread as a function of the opportunity cost of the retained reserve requirement. Therefore, the cost of the reserve requirement rate is passed on to the borrowers, that is, firms.

4.3 Macroprudential policy

The central bank sets monetary policy and macroprudential policy as in the model in section 2.

$$\frac{\mu_t}{\mu} = \left(\frac{D_t}{D_{t-1}}\right)^{\varphi_{d,\mu}} V_{\mu,t} \tag{39}$$

$$\frac{R_t}{R} = \left(\frac{\Pi_{p,t}}{\Pi}\right)^{\phi_{r\pi}} V_{R,t} \tag{40}$$

where R is the steady-state policy rate, Π is the steady-state inflation rate, μ is the steady-state RR rate, $\phi_{r\pi}$ and $\phi_{d\mu}$ measure the elasticity (sensitivity) of the interest rate to deviations in inflation from the steady-state rate and the sensitivity of the RR rate to deviations of credit volume in t to the credit volume in t-1, respectively. $V_{R,t}$ is a monetary policy shock, and $V_{\mu,t}$ is a macroprudential shock.

4.4 Parameter calibration

The parameters of the DSDE model with a banking system are defined according to the permissive range estimated by Bayesian techniques in Schoder (2017a). Table 2 shows the parameters and the assigned values for the model simulation.

Table 2 – Parameter calibration

Parameters	Definition	Value
Household		
Γ	Growth factor of labor-embodied productivity	1.01
β	Household discount rate	0.998
D	Probability of death (inactive households)	0.002
U	Probability of income loss (active households)	0.0006
κ	Steady-state consumption-wealth ratio of inactive households	0.004
N	Aggregate labor supply	0.65376
Firm		
α	Output elasticity of capital	0.28
δ	Rate of capital depreciation	0.025
ϵ	Elasticity of substitution of intermediate goods	3
λ	Target debt-capital ratio	[0.2;0.3;0.4;0.5;0.6;0.7;0.8;0.9]
τ_i	Investment adjustment cost scaling parameter	40
τ_p	Price adjustment cost scaling parameter	50
Central Bank		
$\phi_{r,\pi}$	Inflation elasticity of the interest rate	1.1
Π_p	Target inflation rate of the monetary authority	1
R	Interest target of the monetary authority	1.002
$\tilde{G}$	Steady-state government expenditures	0.2
ρ_G	Persistence of a government spending shock	0.78
μ	Steady-state reserve requirement	0.2
ϕ_μ	Response of the reserve requirement to credit oscillations	0.255
Bargaining problem		
$\phi_{\nu,u}$	Unemployment elasticity of the workers' bargaining power	0 and 1.3
ρ_ν	Persistence of wage inflation	0.4

Source: (SCHODER, 2017a; SCHODER, 2016; VINHADO; DIVINO, 2016)

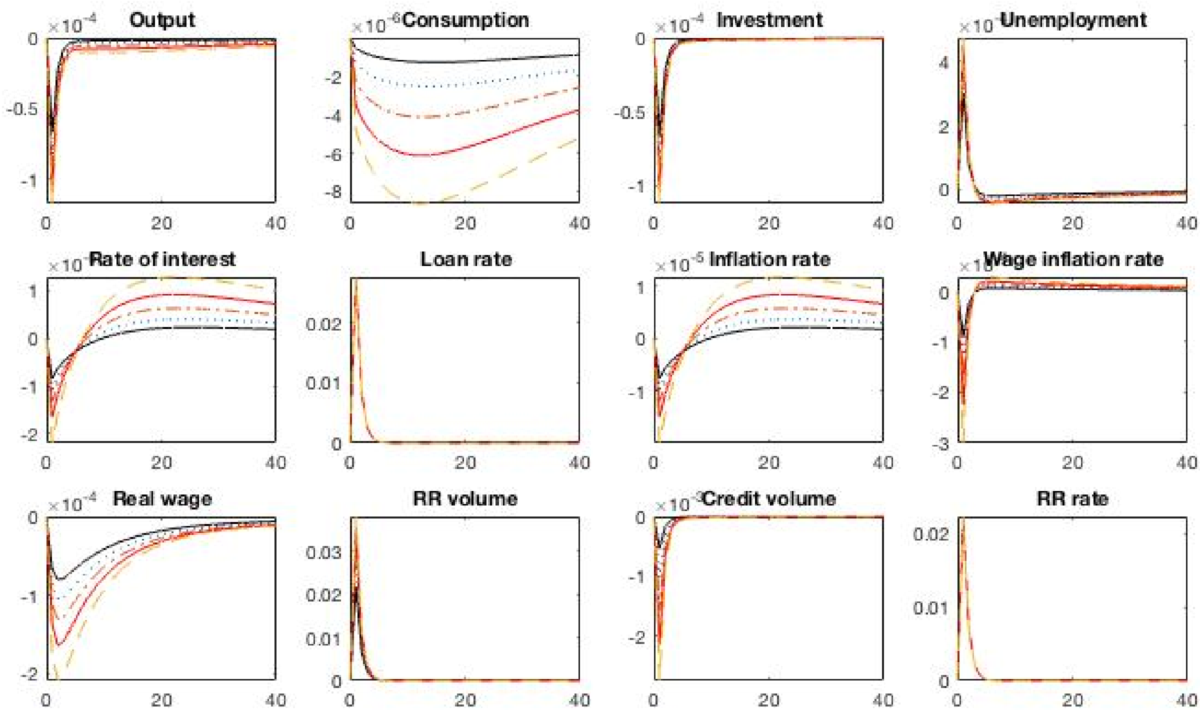
Some parameters maintain the same values because they are within the permissive range of Bayesian estimation such as the inflation elasticity of the interest rate ($\phi_{r,\pi}$) and the probability of death (D). In contrast, the parameters of the adjustment cost scaling parameters for price τ_p and investments

τ_i, as well as the parameters of the persistence of the shock are set according to the estimation results described in Schoder (2017a): $\rho_G = 0.78; \rho_I = 0.7; \rho_\nu = 0.4; \rho_R = 0.8$. The standard deviations of the shock are also calibrated according to the estimation: $\varepsilon_{R,t} = 0.001; \varepsilon_{G,t} = 0.02; \varepsilon_{\nu,t} = 0.005$. Here, $\phi_{\upsilon,u}$ takes values of 0 for no labor market feedback and 1.3 for labor market feedback (given the Bayesian estimation results). The aggregate labor supply (N) is also calibrated such that 10% of the labor force is unemployed in the steady state. The parameter values regarding macroprudential policy take the same values as stipulated in Vinhado and Divino (2016) and in the model in section 2. Finally, the values of the λ parameter are varied to obtain a better understanding of the model and verify the robustness of the results. The remaining parameters are defined as in Schoder (2016).

4.5 Impulse-response analysis

In this section, this simulation exercise was conducted to observe how the model behaves with variation of the lambda parameter value (debt-capital ratio) to evaluate the DSGE model predictions. The impulse-response analysis of macroprudential, fiscal, monetary and worker's bargaining power shocks is depicted in figures 5, 6, 7 and 8, respectively. The solid black line is a simulation with lambda calibrated as 0.2, the dotted blue line is with lambda at 0.3, the orange dash-dot line is with lambda at 0.4, the red solid line is with lambda at 0.5, and finally, the yellow dashed line is with lambda at 0.6. The remaining paremeters are defined as in section 4.4.

Figure 5 – Responses to a 1% persistent reserve requirement shock



Note: the figure shows deviations from the steady state. The model simulation assumes feedback from the labor market to wage formation $\phi_{\nu,u} = 1.3$. The solid black line is the case with with lambda calibrated as 0.2, the dotted blue line is with lambda at 0.3, the orange dash-dot line is with lambda at 0.4, the red solid line is lambda at 0.5, and finally, the yellow dashed line is with lambda at 0.6.

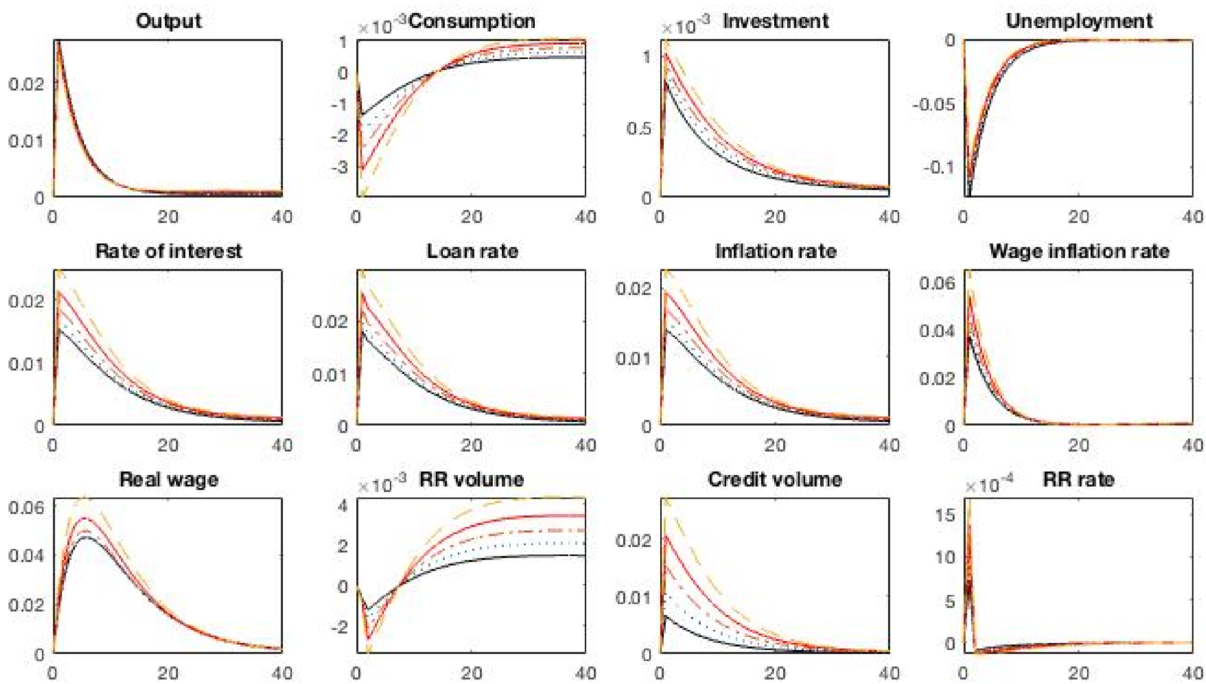
One property of the model is the fixed nominal wage given the bargaining process between worker and firm representatives. The simulation exercise in the last section examined the DSDE model with a constant wage inflation rate, that is, $\phi_{\nu,u} = 0$. Now, the model simulation process assumes feedback from the labor market into wage formation $\phi_{\nu,u} = 1.3$ given the Bayesian estimation in

Schoder (2017a). In general and in all simulations, the variable responses behave in the same manner for each lambda value following each particular shock. The difference is in the magnitudes of the reponses in variable values.

The macroprudential policy shock is displayed in Figure 5. The responses of output, consumption, investment, and credit volume to an RR shock are negative, which is in keeping with economic theory, in all simulations. The consumption variables show some persistence in all simulations, evolving toward a higher magnitude with higher values of lambda. The effects of the shock reduce price inflation, which in turn leads the central bank to lower interest rates. The workers observe the decline in output and consumption prices and accept lower wages.

The fiscal shock changes the behavior of the variables in this simulation (Figure **??**). As the wage inflation rate rises by more than price inflation, real wages increase as the interest rate increases, which has a negative impact on consumption. In contrast, investment, output, and credit volume exhibit the increases expected in a Keynesian model.

Figure 6 – Responses to a 1% persistent government spending shock

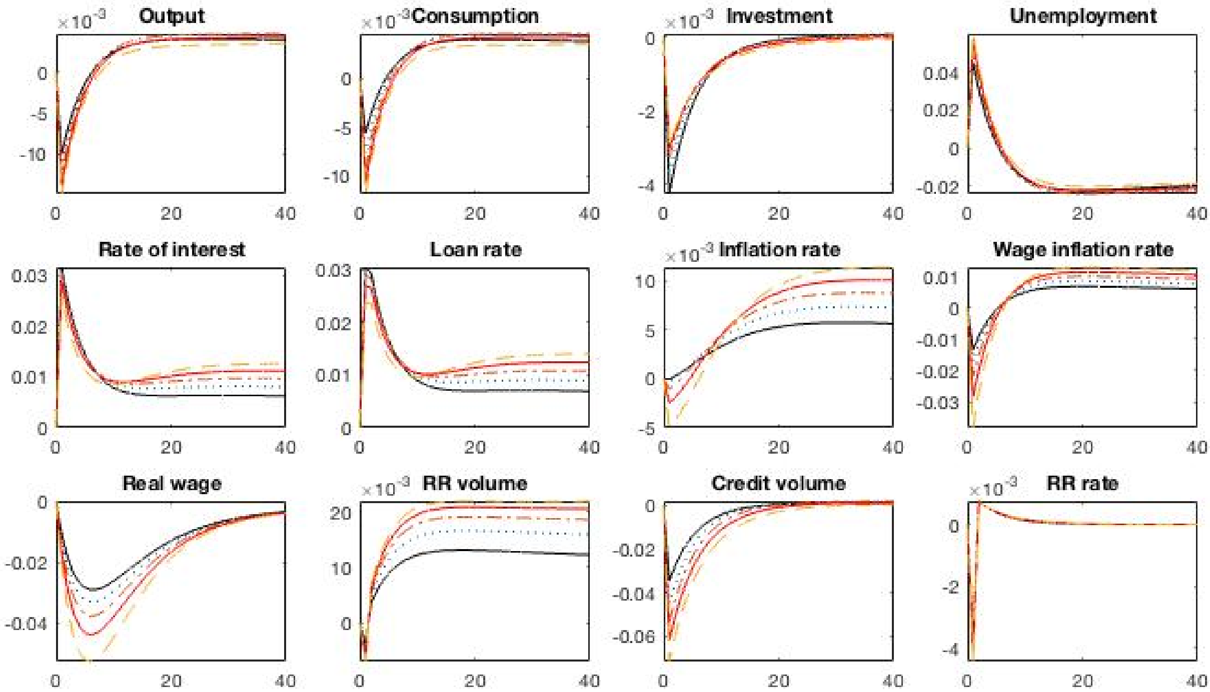


Note: the figure shows deviations from the steady state. The model simulation assumes feedback from the labor market to wage formation $\phi_{\nu,u} = 1.3$. The solid black line is the case with with lambda calibrated as 0.2, the dotted blue line is with lambda at 0.3, the orange dash-dot line is with lambda at 0.4, the red solid line is with lambda at 0.5, and finally, the yellow dashed line is with lambda at 0.6.

A monetary policy shock has the same effect on the majority of variables (Figure 7) as the DSDE model in last section. There are reductions in aggregate demand and credit volume. However, there is a disparity in prices. Price inflation declined by more and reduced real wages, indicating that the decline in wage inflation was steeper than that of price inflation. Thus, the interest rate and loan rate exhibit persistency after ten periods. The shock of introducing persistent worker bargaining power into a model with labor market feedback reduces consumption and output, leading to higher prices, interest rates and real wages; see figure 8.

Another simulation exercise is conducted with the DSDE model while assuming no feedback from the labor market to wage formation $\phi_{\nu,u} = 0$. The lambda variable varies between 0.2 and 0.9. Figures 9, 10, 11 and 12 are the impulses responses and are represented in annex A3. The solid black

Figure 7 – Responses to a 1% persistent interest rate shock



Note: the figure shows deviations from the steady state. The model simulation assumes feedback from the labor market to wage formation $\phi_{\nu,u} = 1.3$. The solid black line is the case with with lambda calibrated as 0.2, the dotted blue line is with lambda at 0.3, the orange dash-dot line is with lambda at 0.4, the red solid line is with lambda at 0.5, and finally, the yellow dashed line is with lambda at 0.6.

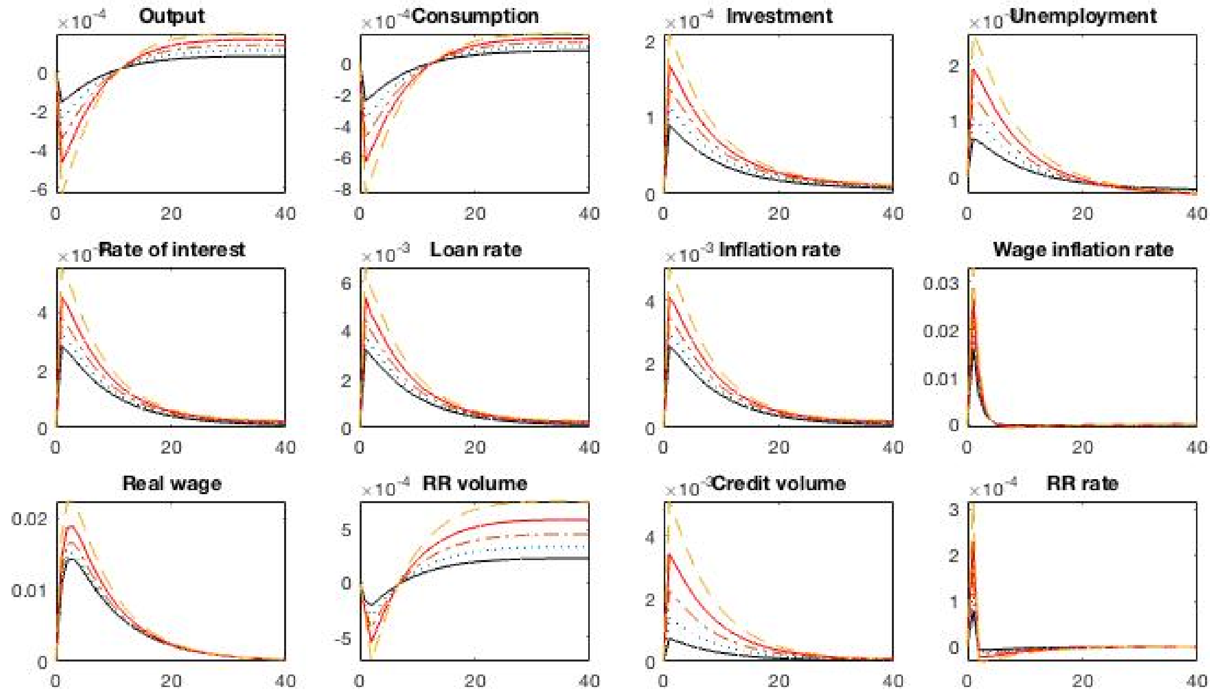
line is the case with with lambda calibrated as 0.2, the dotted blue line is with lambda at 0.3, the orange dash-dot line is with lambda at 0.4, the red solid line is with lambda at 0.5, the yellow dashed line is with lambda at 0.6, the dark blue line is with lambda at 0.7, the green line is with lambda at 0.8, and the light blue line is wit lambda at 0.9. In general, the model behaves in a similar way to the DSDE model calibrated with $\phi_{\nu,u} = 1.3$. The fiscal shock simulation reveals only differences in the consumption variable and real wage (counter-cyclical behavior). Following the monetary shock, the majority of variables exhibit the same patter. The difference is in price variables such as inflation, especially the wage inflation rate and real wage, which exhibit erratic behavior and persistence. The worker bargaining power shock increases the magnitude of the impact of these variables. Finally, the RR shock also exhibits the same pattern in the main variables, albeit less so in the interest and inflation rates.

Conclusion

The overall aim here is to verify the model's response, through key macroeconomic variables, to changes in economic policies and to external shocks. This article seeks to contribute to theoretical modeling for the evaluation of macroeconomic scenarios, particularly for models of dynamic intertemporal optimization, which are widely used by policy makers to understand the Brazilian framework and to forecast economic variables. Specifically, this analysis contributes to the important function of evaluating economic policy strategies and their effects on long-term growth.

The article proposes the inclusion of a macroprudential instrument, an RR that acts directly on household wealth, in a Keynesian framework in which households face the probability of income loss, save in a precautionary way, and are subject to unemployment and wage inflation is fixed by

Figure 8 – Responses to a 1% persistent worker's bargaining power shock



Note: the figure shows deviations from the steady state. The model simulation assumes feedback from the labor market to wage formation $\phi_{\nu,u} = 1.3$. The solid black line is the case with with lambda calibrated as 0.2, the dotted blue line is with lambda at 0.3, the orange dash-dot line is with lambda at 0.4, the red solid line is with lambda at 0.5, and finally, the yellow dashed line is with lambda at 0.6.

Nash solution to a bargaining process between worker and firm representatives. The continuum of intermediate good firms faces quadratic adjustment costs in capital and prices and is subject to some restrictions in its operating environment.

The simulation of the DSDE model with macroprudential policy reveals that, although it is still being calibrated, the model has already demonstrated some expected results. RRs reduce price inflation, credit volume and the interest rate. Furthermore, the RR causes a small recession in the economy, with reductions of output, consumption and investment, but the reduction in aggregate demand is smaller than that from a monetary policy shock. However, one problem was identified: the persistence of some variables after a given shock. Fiscal and monetary policy shocks showed some outcomes predicted by Keynesian models without the crowding-out of private investment given an increase in government spending and a negative impact on aggregate demand given an interest rate shock. This is a peculiarity of disequilibrium models with fixed prices even in the steady state. The bargaining problem between workers and firms over the rate of wage inflation is a important assumption of the DSDE model and is another Keynesian feature.

Based on the model elaborated above, future developments of the model should estimate the parameters from Brazilian data using Bayesian techniques to determine, using the actual values, the structural parameters of the model in the Brazilian context and to fine-tune the model using data. Another possibility is the inclusion of more complex financial intermediation in the financial sector.

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A APPENDIX

A1 Calculation of the Household FOC

Inactive Households:

$$V_i(b_{i,t-s,t-1}) = \max_{c_i,t-s,t} \{lnc_{i,t-s,t-1} + \beta(1-D)E_t V_i(b_{i,t-s,t})\}$$
$$s.t. \quad c_{i,t-s,t} + b_{i,t-s,t} = \frac{R_{t-1}(1+D)}{\Pi_{p,t}} b_{i,t-s,t-1}$$

Substituting out $b_{i,t-s,t}$ and solving the problem:

$$V_i(b_{i,t-s,t-1}) = \max_{c_i,t-s,t} \{lnc_{i,t-s,t-1} + \beta(1-D)E_t V_i(\frac{R_{t-1}(1+D)}{\Pi_{p,t}} b_{i,t-s,t-1} - c_{i,t-s,t})\}$$

The FOC w.r.t. $c_{i,t-s,t}$ implies the optimum consumption for a given state ($b_{i,t-s,t}$):

$$\begin{aligned}\frac{1}{c_{i,t-s,t}} &= \beta(1-D)E_t V_i'(\frac{R_{t-1}(1+D)}{\Pi_{p,t}}b_{i,t-s,t-1} - c_{i,t-s,t}) \\ &= \beta(1-D)E_t V_i'(b_{i,t-s,t-1}) \end{aligned} \tag{41}$$

The optimal consumption is represented by the function $c^*_{i,t-s,t-1}$ (given the state) and substituted into

$$V_i(b_{i,t-s,t-1}) = lnc^*_{i,t-s,t-1} + \beta(1-D)E_t V_i(\frac{R_{t-1}(1+D)}{\Pi_{p,t}}b_{i,t-s,t-1} - c^*_{i,t-s,t})$$

The FOC w.r.t. $b_{i,t-s,t-1}$ and using 41:

$$\begin{aligned}V_i'(b_{i,t-s,t-1}) &= \frac{1}{c_{i,t-s,t}}c^*_{i,t-s,t-1} + \beta(1-D)E_t V_i'(\frac{R_{t-1}(1+D)}{\Pi_{p,t}}b_{i,t-s,t-1} - c^*_{i,t-s,t}) \\ &= \beta(1-D)E_t V_i'(b_{i,t-s,t-1})\frac{R_{t-1}(1+D)}{\Pi_{p,t}} \\ &= \frac{1}{c_{i,t-s,t}}\frac{R_{t-1}(1+D)}{\Pi_{p,t}}\end{aligned}$$

Iterating forward by one period and substituting into 41 yields the Euler equation:

$$\frac{1}{c_{i,t-s,t}} = \beta(1-D)E_t\frac{R_{t-1}(1+D)}{\Pi_{p,t}}\frac{1}{c_{i,t-s,t+1}}$$

Iterating forward by several periods yields

$$\frac{1}{c_{i,t-s,t}} = (\beta(1-D))^n E_t \prod_{k=1}^{n}\frac{R_{t+k-1}(1+D)}{\Pi_{p,t+k}}\frac{1}{c_{i,t-s,t+n}}$$

Considering the beginning-of-period wealth in t for newly inactive households that have not yet participated in the insurance market, $R_{t-1}/\Pi_{p,t}b_{i,t-s,t-1} = R_{t-1}/\Pi_{p,t}b_{a,t-1}$ for s = 0, and for inactive households that have been inactive before, $R_{t-1}(1+D)/\Pi_{p,t}b_{i,t-s,t-1}$ for $s = 1, 2, ..., \infty$. Taking the budget constraint and iterating forward recursively yields[10]

$$\begin{aligned}\frac{R_{t-1}}{\Pi_{p,t}}b_{a,t-1} &= b_{i,t,t} + c_{i,t,t} \\ &= \left(\frac{R_{t-1}(1+D)}{\Pi_{p,t+1}}\right)^{-1} b_{i,t,t+1} + \left(\frac{R_{t-1}(1+D)}{\Pi_{p,t+1}}\right)^{-1} c_{i,t,t+1} + c_{i,t-s,t} \\ &= \sum_{n=0}^{\infty}\prod_{k=1}^{n}\left(\frac{R_{t+k-1}(1+D)}{\Pi_{p,t+k}}\right)^{-1} c_{i,t,t+n} \\ &= \sum_{n=0}^{\infty}\prod_{k=1}^{n}\left(\frac{R_{t+k-1}(1+D)}{\Pi_{p,t+k}}\right)^{-1} c_{i,t,t+n}(\beta(1-D))^n E_t\prod_{k=1}^{n}\frac{R_{t+k-1}(1+D)}{\Pi_{p,t+k}}\frac{1}{c_{i,t,t+n}}c_{i,t,t}\end{aligned}$$

[10]
$$b_{i,t-s,t-1} = \left(\frac{R_{t-1}(1+D)}{\Pi_{p,t}}\right)^{-1} b_{i,t-s,t} + \left(\frac{R_{t-1}(1+D)}{\Pi_{p,t}}\right)^{-1} c_{i,t-s,t+1}$$

$$
\begin{aligned}
&= \sum_{n=0}^{\infty}(\beta(1-D))^n c_{i,t,t} \\
&= \frac{1}{1-\beta(1-D)} c_{i,t,t} \\
c_{i,t,t} &= (1-\beta(1-D))\frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1} \\
c_{i,t,t} &= \kappa \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1}
\end{aligned}
$$

It follows that the inactive household chooses consumption proportional to its previous wealth. We deduce the same relationsships of consumption and wealth for inactive households that have been inactive before ($R_{t-1}(1+D)/\Pi_{p,t} b_{i,t-s,t-1}$ for $s = 1, 2...\infty$):

$$c_{i,t-s,t} = \kappa(1+D)\frac{R_{t-1}}{\Pi_{p,t}} b_{i,t-s,t-1}$$

Active household problem:

$$
\begin{aligned}
V_a(b_{a,t-1}) &= \max_{c_{a,t}}\{lnc_{a,t-1} + \beta(1-U)E_t V_a(b_{a,t}) + \beta U E_t V_i(b_{a,t})\} \\
s.t. \quad c_{a,t} + b_{a,t} &= \omega(1-u_t)n + \pi_{d,t} - t_t - \tau_t + \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1} - \mu_t b_{a,t}
\end{aligned}
$$

The FOC w.r.t. consumption is

$$\frac{1}{c_{a,t}} = \beta(1-U)E_t V_a'(b_{a,t}) + \beta U E_t V_i'(b_{a,t})$$

Substituting the function $c_a^*(b_{a,t-1})$ back into the objective (given state, $b_{a,t-1}$),

$$V_a(b_{a,t-1}) = \max_{c_{a,t}}\{lnc^*_{a,t-1}(b_{a,t-1}) + \beta(1-U)E_t V_a(b_{a,t}) + \beta U E_t V_i(b_{a,t})\}$$

The FOC w.r.t. $b_{a,t-1}$, noting that $c_a^{*\prime}(b_{a,t-1}) = 0$ and using the FOC w.r.t. consumption, yields

$$
\begin{aligned}
V_a'(b_{a,t-1}) &= \frac{1}{c_a^*(b_{a,t-1})} c_a^{*\prime}(b_{a,t-1}) + \beta(1-U)E_t V_a'(b_{a,t})\left(\frac{R_{t-1}}{\Pi_{p,t}} * \frac{1}{1+\mu_t} - c_a^{*\prime}(b_{a,t-1}) * \frac{1}{1+\mu_t}\right) + \\
&\quad + \beta U E_t V_i'(b_{a,t})\left(\frac{R_{t-1}}{\Pi_{p,t}} * \frac{1}{1+\mu_t} - c_a^{*\prime}(b_{a,t-1}) * \frac{1}{1+\mu_t}\right) \\
&= \beta(1-U)E_t V_a'(b_{a,t})\frac{R_{t-1}}{\Pi_{p,t}}\frac{1}{1+\mu_t} + \beta U E_t V_i'(b_{a,t})\frac{R_{t-1}}{\Pi_{p,t}}\frac{1}{1+\mu_t} \\
&= \frac{1}{c_{a,t}}\frac{R_{t-1}}{\Pi_{p,t}}\frac{1}{1+\mu_t}
\end{aligned}
$$

A2 Firm Aggregation

The law of motion of the capital stock:

$$k_t = i_t + (1-\delta)k_{t-1}$$

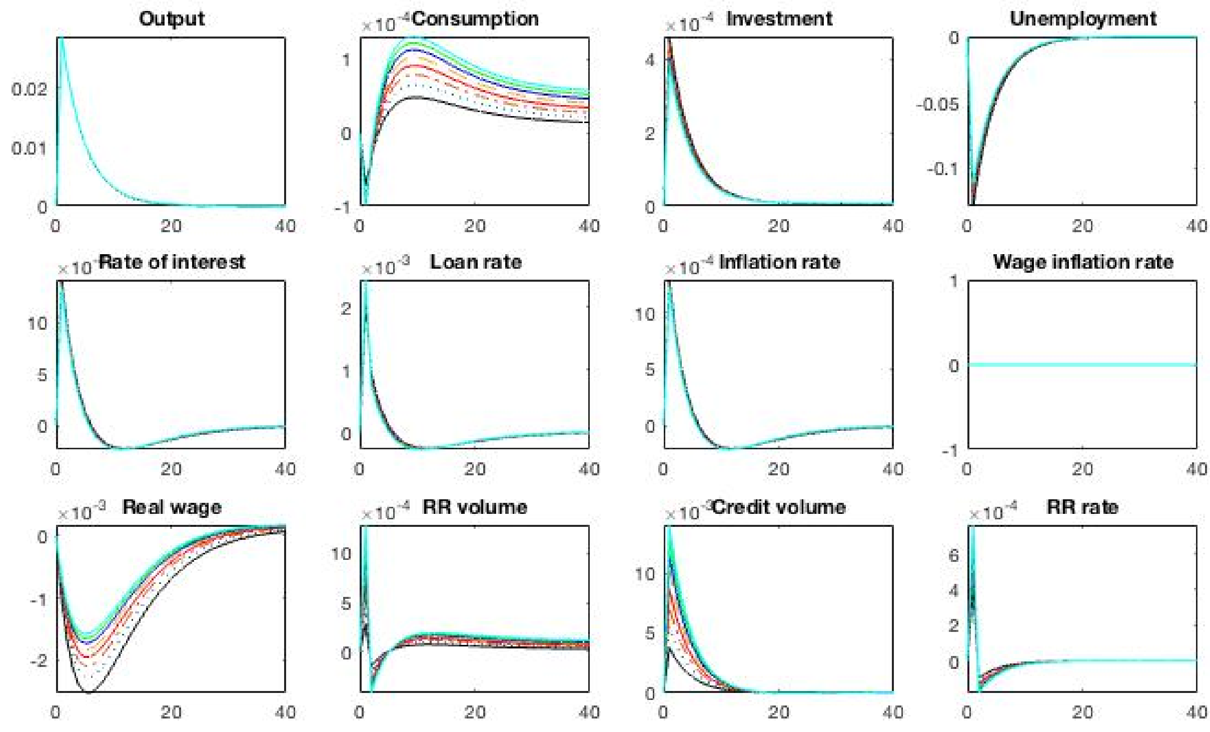
$$K_t = I_t + (1-\delta)K_{t-1}$$
$$\frac{K_t}{\Gamma^t} = \frac{I_t}{\Gamma^t} + (1-\delta)\frac{k_{t-1}}{\Gamma^t}$$
$$\tilde{K}_t = \tilde{I}_t + (1-\delta)\frac{1}{\Gamma}\tilde{K}_{t-1} \tag{42}$$

Production function:

$$y_t = (\Gamma k_{t-1})^{\alpha}(\Gamma^t l_t)^{1-\alpha}$$
$$\left(\frac{p_t}{P_t}\right)^{-\epsilon} Y_t = (\Gamma k_{t-1})^{\alpha}(\Gamma^t l_t)^{1-\alpha}$$
$$Y_t = (\Gamma K_{t-1})^{\alpha}(\Gamma^t L_t)^{1-\alpha}$$
$$\frac{Y_t}{\Gamma^t} = \left(\frac{\Gamma K_{t-1}}{\Gamma^t}\right)^{\alpha}(L_t)^{1-\alpha}$$
$$\tilde{Y}_t = (\tilde{K}_{t-1})^{\alpha}(L_t)^{1-\alpha} \tag{43}$$

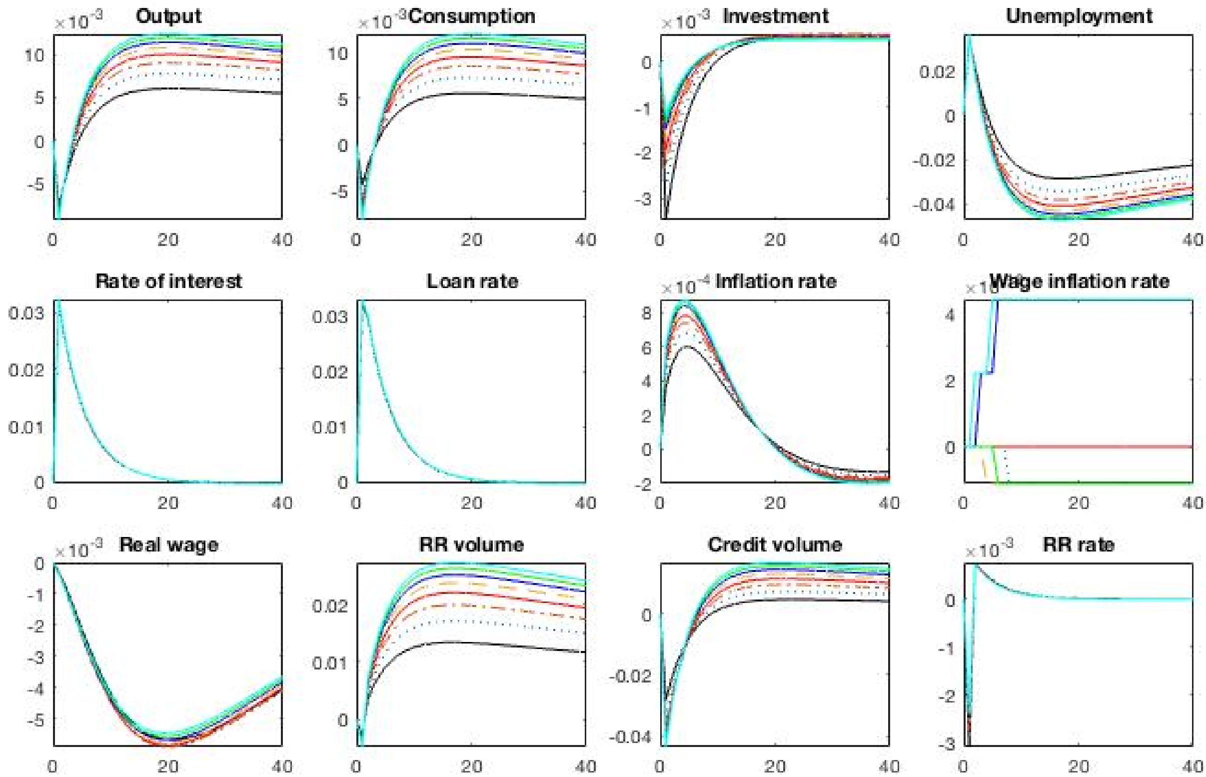
A3 Impulse responses

Figure 9 – Responses to a 1% persistent government spending shock



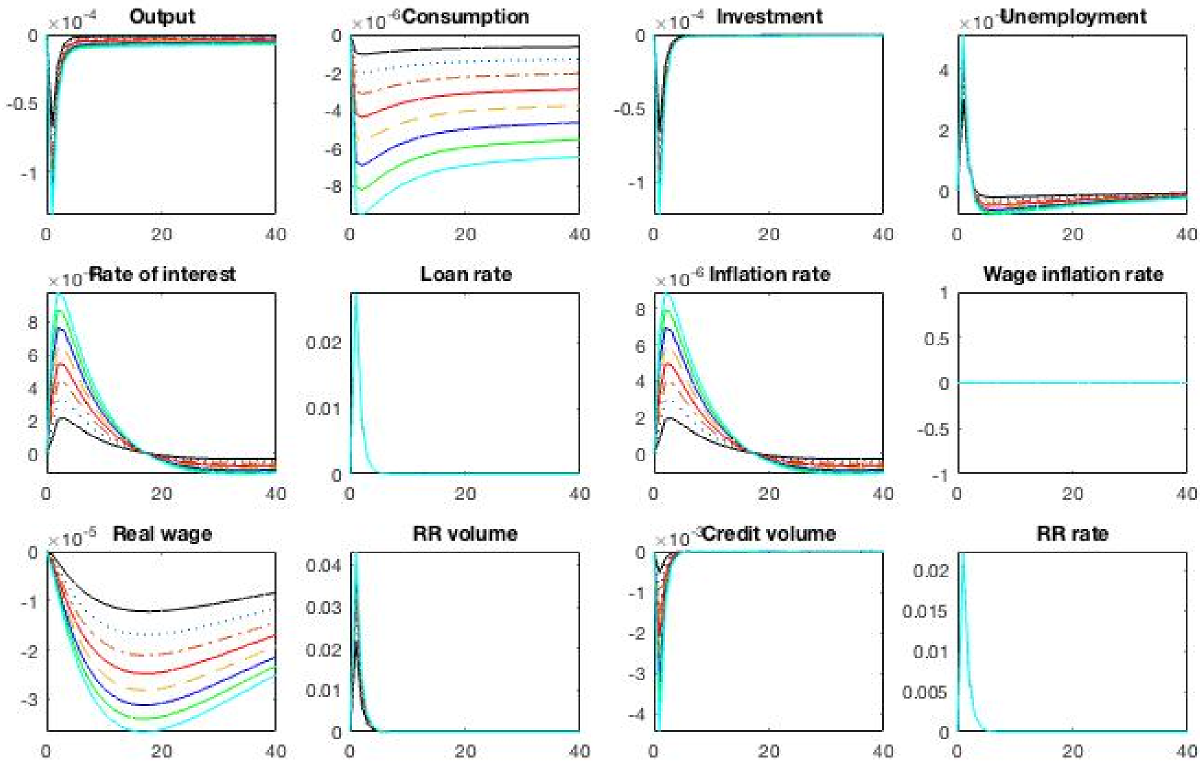
Note: This simulation exercise assumes no feedback from the labor market to wage formation $\phi_{\nu,u} = 0$. The solid black line is the case with with lambda calibrated as 0.2, the dotted blue line is with lambda at 0.3, the orange dash-dot line is with lambda at 0.4, the red solid line is lambda at 0.5, the yellow dashed line is with lambda at 0.6, the dark blue line is with lambda at 0.7, the green line is with lambda at 0.8, and the light blue line is with lambda at 0.9. In general, the model behaves in similarly to the DSDE model calibrated with $\phi_{\nu,u} = 1.3$.

Figure 10 – Responses to a 1% persistent interest rate shock



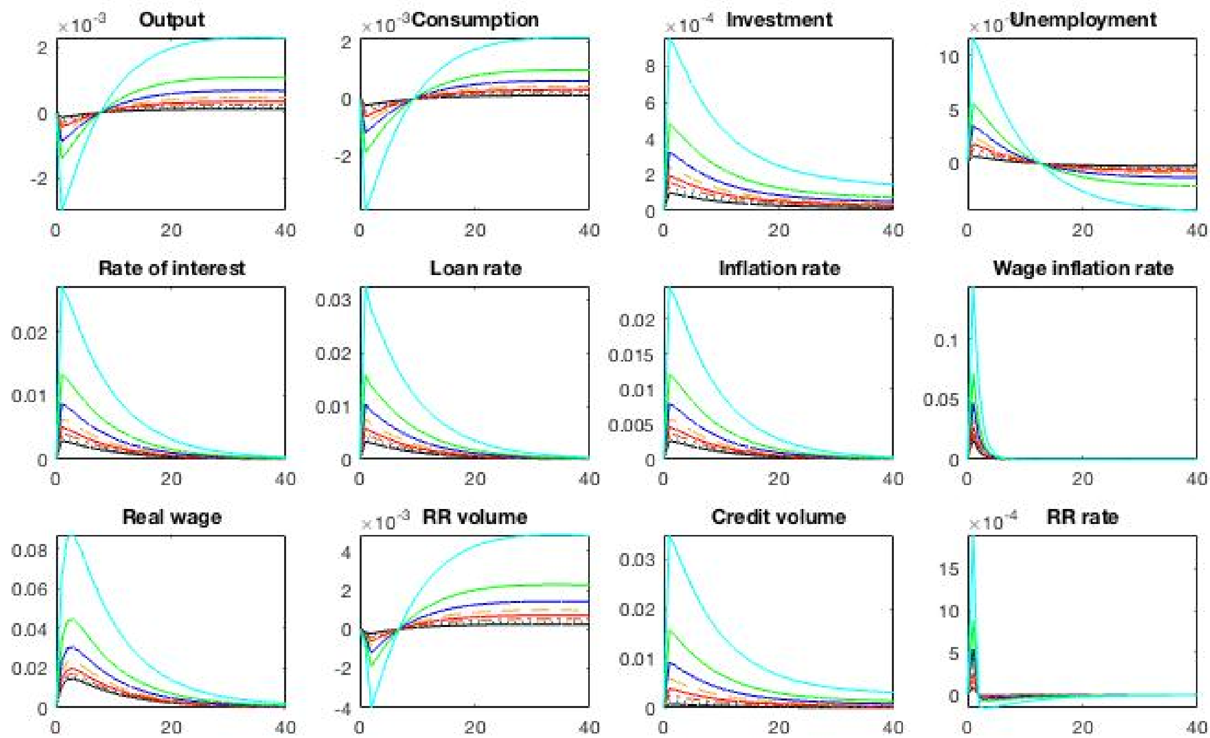
Note: This simulation exercise assumes no feedback from the labor market to wage formation $\phi_{\nu,u} = 0$. The solid black line is the case with with lambda calibrated as 0.2, the dotted blue line is with lambda at 0.3, the orange dash-dot line is with lambda at 0.4, the red solid line is with lambda at 0.5, the yellow dashed line is with lambda at 0.6, the dark blue line is with lambda at 0.7, the green line is with lambda at 0.8, and the light blue line is with lambda at 0.9. In general, the model behaves similarly to the DSDE model calibrated with $\phi_{\nu,u} = 1.3$.

Figure 11 – Responses to a 1% persistent reserve requirement shock



Note: This simulation exercise assumes no feedback from the labor market to wage formation $\phi_{\nu,u} = 0$. The solid black line is the case with with lambda calibrated as 0.2, the dotted blue line is with lambda at 0.3, the orange dash-dot line is with lambda at 0.4, the red solid line is with lambda at 0.5, the yellow dashed line is with lambda at 0.6, the dark blue line is with lambda at 0.7, the green line is with lambda at 0.8, and the light blue line is with lambda at 0.9. In general, the model behaves similarly to the DSDE model calibrated with $\phi_{\nu,u} = 1.3$.

Figure 12 – Responses to a 1% persistent worker's bargaining power shock



Note: This simulation exercise assumes no feedback from the labor market to wage formation $\phi_{\nu,u} = 0$. The solid black line is the case with with lambda calibrated as 0.2, the dotted blue line is with lambda at 0.3, the orange dash-dot line is with lambda at 0.4, the red solid line is with lambda at 0.5, the yellow dashed line is with lambda at 0.6, the dark blue line is with lambda at 0.7, the green line is with lambda at 0.8, and the light blue line is with lambda at 0.9. In general, the model behaves similarly to the DSDE model calibrated with $\phi_{\nu,u} = 1.3$.