Income Inequality, Over-indebtedness and Financial Instability
Essay on a Keynes-Goodwin-Minsky Macrodynamic Model

Elom K. EZUHO
PhD student in Development Economics at CERDI – Université d'Auvergne
65 bd François Mitterrand, 63000 Clermont-Ferrand, France
E-mail: Komi_Elom.Ezuho@u-clermont1.fr

October, 14th 2011

Abstract

This paper aims at modeling the impact of income inequality on financial instability through the channel of over-indebtedness.
The paper develops a macrodynamic model based on the first hand, on Goodwin (1967) model inspired by the Lotka-Volterra predator-prey equations and in which debt is introduced, and on the second hand, on Minsky (1975, 1986) financial instability hypothesis as predator-prey struggle. The whole relies on Keynes theoretical elements to give the Keynes-Goodwin-Minsky model. The model differs from the precedent Keynes-Goodwin and financial instability models by two new characteristics. First it considers directly a measure of income inequality as a dynamic state variable. Second it takes simultaneously into account households' debt and firms' debt ratios without simplification assumptions. The model is a four-dimensional dynamic system with four state variables which are Gini coefficient, households' debt ratio, firms' debt ratio and the aggregate demand. It considers three economic agents: households (workers), firms and rentiers (considered as financial institutions). Using Routh-Hurwitz conditions of local stability, the model is proved to be locally stable. The phase diagram shows a cycle of macro-financial instability primarily induced by income distribution disequilibrium.

Key words: income inequality, aggregate demand, over-indebtedness, financial instability

1. INTRODUCTION

After the recent crisis burst, many causes and explanations emerged but few of them pointed out income inequality widespread as a mainstream cause. Fitoussi and Saraceno (2010) argued that although the crisis may have emerged in the financial sector, its roots are much deeper and lie in a structural change in income distribution that has been going on for the past three decades. Analyzing data on income shares and bank failures in the case of the United States, David A. Moss (2010) was surprised by the tight correlation between these two variables before and during the 2007 crisis as well as for the 1929's one. When income inequality increases, households get more and more into debt in order to maintain their living standards. This reaction is amplified by the so-called *social demonstration effect* of Duesenberry (1949) and a wide financial deregulation. The search for high-return investment by those who benefited from the increase in inequalities led to the emergence of bubbles. Net wealth became overvalued, and high asset prices gave the false impression that high levels of debt were sustainable (Fitoussi and Saraceno, 2010). Unsustainable debt leads to reimbursement default and to financial institutions failures (Stiglitz, 1969). Is income inequality and financial crisis relation a simple correlation, a common causation (e.g.: neoliberal ideology) or an actual causation where inequality creates a macroeconomic vulnerability (Krugman, 2010)? Income inequality and private debt are linked (Iacoviello, 2007), such as over-indebtedness and financial crisis (Fisher, 1933; Minsky, 1975, 1977, 1986). But at our knowledge, there are not yet relevant studies, empirical or theoretical, relating income inequality to financial instability.

It's often seems more intuitively logic that income inequality increasing can be a consequence of financial crises while the reverse causality, i.e income inequality as a cause of financial crises, doesn't bring unanimity among economists. As Fisher and Minsky's theories had not been modeled by the authors, further theoretical models had been developed from them (Keen S., 1995; Keen S., 2000; Taylor and O'Connell, 1985; Semmler, 1987; Franke and Semmler, 1989; Franke and Asada, 1994; Delli Gatti and Gallegati, 1995; Asada, 2001; Asada 2006). Albeit all these models pointed income distribution out as an important bone in the financial instability dynamics, they did not consider it as a direct state variable. Furthermore, while some of these models dwell on firms' debt, disregarding households' debt, with sole simplification matter, the others did the contrary. In fact, these lacks don't strengthen the capacity of these models to digging up the core cause of financial crisis. To compensate for these lacks might then offer a serious issue of understanding better, financial crisis. This paper aims at modeling the impact of income inequality on financial instability through the channel of over-indebtedness.

The paper develops a macrodynamic model based on the first hand, on Goodwin (1967) model inspired by the Lotka-Volterra[1] predator-prey equations and in which debt is introduced, and on the second hand, on Minsky (1975, 1986) *financial instability hypothesis* as predator-prey struggle. The whole relies on Keynes theoretical elements to give the Keynes-Goodwin-Minsky model. The model differs from the precedent Keynes-Goodwin and financial instability models by two new characteristics, compensating for the lacks pointed out above. Firstly it considers directly a measure of income inequality as a dynamic state variable. Secondly it takes simultaneously into account households' debt and firms' debt without simplification assumptions which tend implicitly to disregard the one or the other. The model is a four-dimensional dynamic system with four state variables which are Gini coefficient, households' debt, firms' debt and the aggregate demand. It considers three economic agents: households (workers), firms and rentiers (considered as financial institutions). Using Routh-Hurwitz conditions of local stability, the model is proved to be locally stable. The phase diagram shows a cycle of macro-financial instability primarily induced by income distribution disequilibrium.

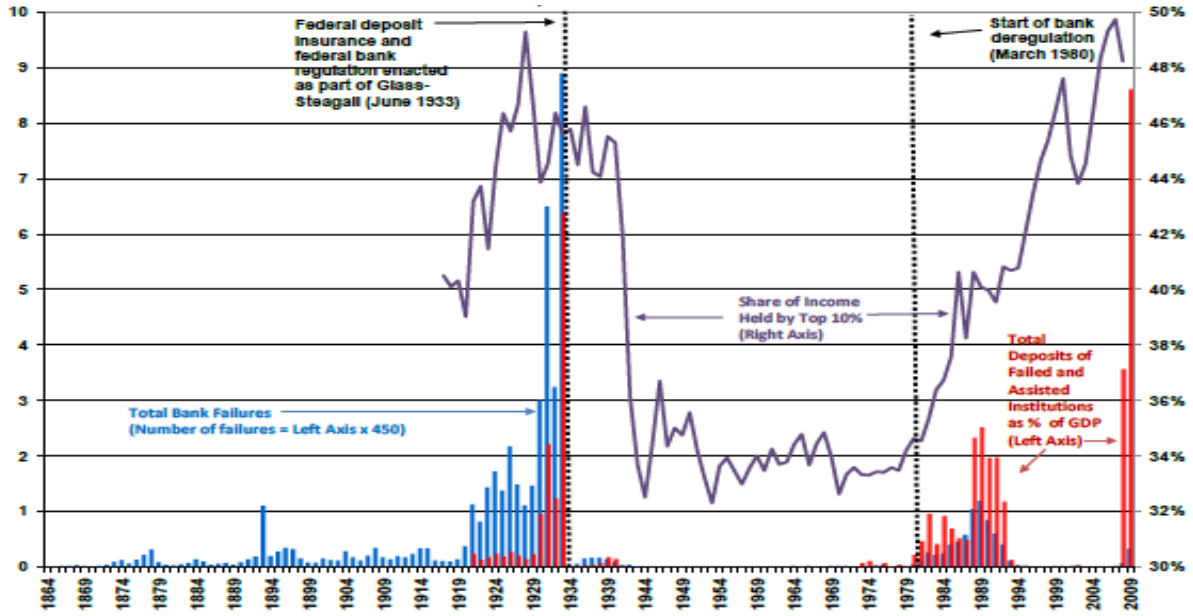
The remainder of the paper is structured as follows. Section 2 reveals some empirical facts on the correlation between income inequality and financial crisis. Sections 3 and 4, focus on the main theoretical literature, the first on income inequality and debt and the later on debt and financial crisis with their critics. While section 5 presents the Keynes-Goodwin-Minsky model, section 6 deals with the stability proof of the model using Routh-Hurwitz conditions. Economic interpretations are presented in section 7 before the conclusion.

2. INCOME INEQUALITY AND FINANCIAL CRISIS: SOME EMPIRICAL FACTS

Despite most economists are skeptic about a real causation between income inequality and financial crisis, some empirical observations, show doubtless their tight correlation. Let's consider two cases: USA and UK, because of data availability and the fact that the 2007 crisis had been triggered there first.
Figure 1 shows (USA case) income inequality variations as share of income held by top 10% compared to bank failures and total deposits of failed and assisted institutions (as % of GDP). Income disparities before the two great depressions of 1929 and 2007-2008 have been the most important over the last century with a remarkable similarity.

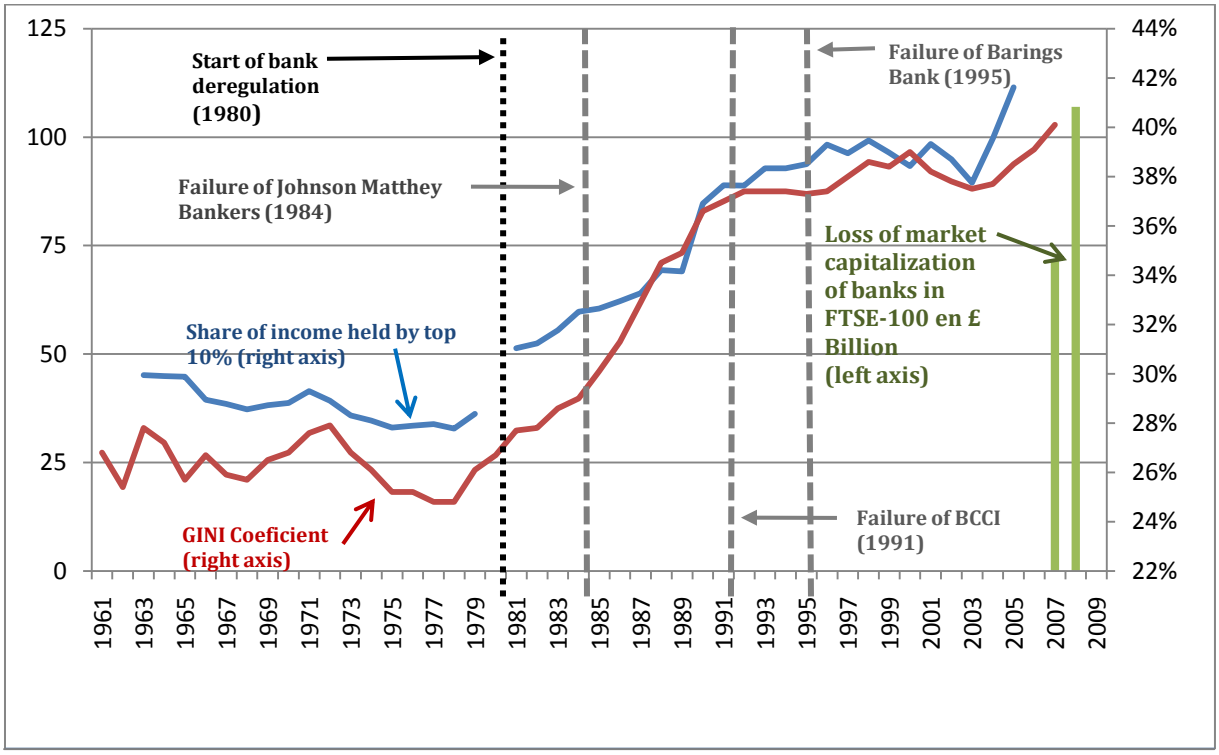
[1] **Lotka A. J.** (1925) "Elements of physical biology", Baltimore:Williams and Wilkins Co; and **Volterra V.** (1926) "Variazioni e fluttuazioni del numero d'individui in specie animali conviventi", Mem. R. Accad. Naz. Dei Lincei, VI, 2; figured out, independently from each other the first mathematical model for biological systems based on fish in the Adriatic and it's main predator.

Fig 1: Bank failures, financial regulation and income inequality in USA (1872-2009)



Source: David A. Moss, August 2010

Fig. 2: Bank failures, financial regulation and income inequality in UK (1961-2008)



Data Source: Share of income held by top 10% (Atkinson A., Piketty T., and Saez E. « Top Incomes in the Long Run History », Journal of Economic Littérature, 2010) ; GINI coeficient: from 1961 to 1993 Family Expenditure Survey, from 1994 to 2008 Family Ressources Survey (Townsend I., Economic Policy and Statistics, Library-House of Commons, SN/EP/3870, July 2009, UK). Bank failures (Reinhart et Rogoff, « This time is different : eight centuries of financial folly » (french version), PEARSON, Octobre 2010) ; Loss of market capitalization of banks in FTSE-100 (Data from Financial Times in Treasury Committee-House of Commons, « Banking crises : dealing with the failure of UK banks », Seventh report of Session 2008-2009, UK).

Following a steady rising, income share held by the top 10% had reached 49.29% and 49.74% of total income respectively in 1928 and 2007, in a financial deregulation context. As shown on fig. 1, the vertical dotted lines mark the bank regulation period from June 1933 to March 1980 where income inequalities dropped drastically and stayed the lowest. It seems important to mention that after 1929's crisis burst, income disparities were still higher a period of time before dropping drastically (from 1939).

Like USA, bank deregulation started from 1980 in UK as well. Income inequalities, lower and quite stable before 1980, began rising steadily while UK had been encountering three huge bank failures respectively in 1984, 1991 and 1995. In 1998, income inequality as share of income held by top 10% and as Gini coefficient reached respectively for each measure 39.74 % of total income and 38.2% comparing to 28.37% and 26% in 1979. Income disparities stayed higher and quite stable at 38.5% before another steady rising began from 2003 until the crisis burst in 2007 causing more than 105 £ billions losses of the market capitalization of bank in the FTSE-100. The share of income held by top 10% in 2007 was up to reach 42% and 40% for the Gini coefficient.

These empirical observations point out two major facts. Firstly, they clearly show how income inequality increasing, generally leads up to financial crisis and great depression. Secondly, after a great depression, income inequality remains higher a period of time before dropping due to recovery measures. This fact is to call into question the above mentioned intuition of income inequality increasing as a consequence of financial crisis or great depression. Then one might ask if the income inequality increasing hype observed during and just after a financial crisis is *or* is not a simple continuity whose origin upstream may be the real cause of the financial crisis instead?

3. INCOME INEQUALITY, BANK DEREGULATION AND HOUSEHOLD DEBT

One of the consequences of higher incomes rapid growth at the expense of lower incomes is the fall of relative purchasing power of the later. It's well known that such a purchasing power fall, of economic agents with higher marginal propensity to consume, has dramatic consequence on aggregate demand and thus on economic dynamism. But confronted with a fall of purchasing power, low and middle income households struggled to maintain their living standards as long as possible.

In fact, consumption doesn't merely depend on the absolute level of current income, but also on current income relative to past income; people whose incomes are low relative to their past current incomes reduce saving and incur deficit, if they have the necessary assets or credit, to protect their living standards (Duesenberry, 1949). Beyond the fact of maintaining their living standards, low and middle income households wish to keep them upward too, as they faced at the same time with a social pressure due to perceptible inequalities in the standards of consumption against higher incomes households. These efforts among households to maintain acquired positions in the social ranking of living standards are essentially based on the 'social visibility' of consumption. That is, this social pressure, more perceptible in mass consumption societies, is among other reasons due to the availability of new goods and services and the publicity hype which makes them "irresistible" so that lower income households would have to incurred deficits to get them (Barba and Pivetti, 2009). Pursuing their analysis, Barba and Pivetti (2009) argued that this kind of fact noticed reveals a tendency of consumption spending to rise even when individual incomes stagnate, provided that household *aggregate* income keeps on rising. So, as income inequalities rise, consumption inequalities seem not to rise as well, at least not proportionally. As a result in a context of financial deregulation, savings rate decreases while households' debt rate rises, followed by a remarkable shift in bank loans policies from business activities to household credits. This inelasticity of consumption inequalities according to income

inequalities (Krueger and Perri, 2005) provides then explanation on rising household debt source. That is, households benefited from credit to maintain and/or to keep upward their living standards such as the economy to avoid aggregate demand depression. And as credit contraction depresses aggregate demand (Bernanke, 1983), bank deregulation has been set up as a must in other to open up access to credit and to sustain the economy.

To sum up, the increase of income inequalities induces two consecutive effects. On the first hand, through the fall of purchasing power of households with higher marginal propensity to consume, it makes aggregate demand depressed such as the whole economy. On the second hand, through the so-called *social demonstration effect* of Duesenberry in a context of less credit regulation, credit rate and households' debt rate increase. These two consecutive effects occurred instantly and systematically one after other and are complementary so that only the second effect of credit rate and households' debt rate increase is really observed a priori. Households' debt increase is the counterpart of their purchasing power fall, kept up thanks to credits, which fill in automatically the shortfall in aggregate demand so as to keep up the economy dynamism as well. But as the leverage effect of credit on aggregate demand is not permanent, the economy, in other to keep up at a full employment level, would need to be fed by credit every time its aggregate demand seems to drop. So, in face of persistent income inequalities, credit turns as the economy's bellows. But is this mechanism indefinitely sustainable?

In fact, credit policy is an inter-temporal mechanism of an initial increase and a post-period decrease of purchasing power. Bank loans to households consist in maintaining or raising in a first-time period the consumption capacity of subscribers (households) who are linked by a reimbursement commitment. The reimbursement represents in the second-time period it has been ongoing, a compensatory fall of the first-time period purchasing power increase effect. And yet at the end, according to economic conditions (variable interest rate), the cumulated effect of the compensatory fall in the purchasing power due to reimbursement can surpass the inductive effect of its first-time period increase, with the difference going to lenders as financial profits. As this difference increases, lower incomes households (credit subscribers) purchasing power dwindles away in favor of lenders who, often, are the higher incomes people. Then, income disparities worsen. Two effects come out. Firstly, income inequality worsening makes lower income households get more and more into debt because efforts to keep up the standards of living become more and more costly as well. Secondly, creditors increase credit supply with more flexible conditions so as to make sure higher future profits. What Stiglitz (2010) called predator credit. The flexible conditions make new soft credits instruments and incite individuals to subscribe again albeit the weight of anterior credits and reimbursement defaults. Fitoussi and Saraceno (2010) highlighted with precision this second effect when arguing "that the search for high-return investment by those who benefited from the increase in inequalities led to the emergence of bubbles. Net wealth became overvalued, and high asset prices gave the false impression that high levels of debt were sustainable". A priori, the economy seems to do well and is even boosted as credit or households' debt sustains aggregate demand. But as longer as the aggregate demand is supported by loans, its sustainability is on a knife edge yielding an "unsustainable sustainability". Finally at some level of excessive debt, the whole economy suddenly breaks down.

4. OVER-INDEBTEDNESS AND FINANCIAL INSTABILITY

The first theoretical essays on financial crises drew inspiration from the great depression of 1929 though many other financial crises or economic depressions occurred before. Two among them retain our attention: the debt-deflation theory of Fisher (1933) and the financial instability hypothesis of Minsky (1975, 1977, 1986).

4.1 THE DEBT-DEFLATION THEORY OF IRVING FISHER

According to Fisher (1933), if debt and deflation are absent, other disturbances are powerless to bring on crises comparable in severity to those of 1837, 1873 or 1929-1933. Over-indebtedness and deflation combination is the principal cause of great depression. In Fisher's theory, over-indebtedness results in over-investment due to over-speculation of future profits while interest rate is low increasing the incentive to borrow and to speculate. The temptation of large future profits and low interest rate fuel credit expansion and give way to speculation. Fisher assumed, accordingly, that at some point of time, a state of over-indebtedness exists. This will tend to lead to liquidation, through the alarm either of debtors or creditors or both. From then on, Fisher deduced a following chain of consequences in nine links. Debt liquidation leads to distress selling and to contraction of deposit currency, as bank loans are paid off, and to a slowing down of velocity of money causing a fall in the level of prices and a still greater fall in the net worth of business precipitating bankruptcies and a like fall in profits. This will lead the concerns – firm[2] – which are running at a loss to make: a reduction in output, in trade and in employment of labor. Pessimism runs through the economy and leads to hoarding and slowing down still more the velocity of money. Then complicated disturbances appear, particularly, a fall in the nominal rates and a rise in the real rates of interests. People try to pay off their debt but the more the debt grows. Because every effort made to lessen the burden of debts increases the real interest rate. Over-indebtedness and deflation act and react on each other until the collapse of the economy.

Fisher's theoretical framework is based on an assumption of initial over-indebtedness. As mentioned above, the over-indebtedness is due to speculation of future profits. Without providing solid theoretic arguments to explain over-indebtedness origin, Fisher mentioned that it may come from the anticipation of new exceptional high-return investment opportunities. Meanwhile, Fisher highlighted cautiously that over-investment and over-speculation are often important for an economy; but they would have far less serious results were they not conducted with borrowed money.

Fisher was still modest in his demonstration and clearly stated that his work is a first step towards more research in this area. First of all, debt-deflation theory starts with the assumption of over-indebtedness without laying out full theoretical details on its beginning. On the second hand, the link the theory describes between deflation and depression seems empirically weak. Atkeson and Kehoe (2004) find that there is no evidence of significant correlation between deflation and depression. Depression is defined as a negative increase of GDP. The authors find out that, although deflation and depression might be correlated in the 1930's, there is nothing significant for 17 other developed countries on more than a century period of time. For Bernanke and Tobin, debt-deflation theory must be viewed primarily as a propagation mechanism (Von Peter, 2005) in contrast with Minsky (1986) who considered it as a fully endogenous phenomenon, not merely a mechanism that propagate external shocks. For this purpose, Minsky went beyond Fisher and tried to provide a more significant theoretical contribution to financial instability analysis.

4.2. THE FINANCIAL INSTABILITY HYPOTHESIS OF HYMAN MINSKY

Minsky's ideas concur with Fisher's in a trilogy described by "over-speculation, over-investment and over-indebtedness". Drawing inspiration not only from Fisher, but also from Kalecki and much more from Keynes, Minsky's (1977) contribution originality consisted in a theory, so-called *financial instability hypothesis*, of how the normal functioning of financial markets endogenously generated by a capitalist economy gives rise to financial crises.

[2] Fisher used the expression of « private-profit society »

The *financial instability hypothesis* starts from a normal economic situation close to full employment which incites economic agents to anticipate positive high-return investments. As insurance of holding money fell, economic agents clearly showed a preference for holding capital assets through three types of financing, different in the way of commitments and the returns they would yield. Agents are offered "hedge financing", "speculative financing" or "Ponzi financing" possibilities. The first is based on commitments to make future payments covered by a certain income while the second is based on commitments to make future payments which may or may not be covered by future income. The third one relies on commitments to make future payments which can only be covered by issuing new liabilities, such as borrowing. As it can be easily noticed, hedge financing possibility is associated to almost no risk, the speculative financing structures to some level of risk and the Ponzi's to the highest. It also appeared that the riskiest is the financing structure the highest would be its returns. And as Minsky suggested, profits are the key to a capitalist economy; they determine present and future investment as well as validating past investment decisions. Anticipating high-returns, agents would run to make profits. Unsurprisingly, investment demand will increase and will lead to an increase in profits which, in turn, will give steady rise to capital assets' price. The circle then formed leads to further increase of investment demand, profits and price of capital asset and so on. The economy is boosted. That is what usually called a boom.

Drastic increase in investment demand against investment supply which cannot follow the same movement leads to a rapid increase in short-run interest rates and further in the long-run rates. Then, value of profits will decrease, followed tightly by a decrease in investment as well and furthermore an overall decrease in value of profits. Profits decrease will lead to negative expectations, a fall in the insurance of holding capital assets and their distress selling. The above mentioned circle of economic boom rapidly, turns to a vicious circle of depression. Decrease in profits and distress selling lead to a fall in the price of capital assets, investment, profits and so on. With profits fall comes an inability to fulfill financial commitments and agents go much more into debt with increasing speculative and Ponzi financing structure while hedge financing ones are flowing down. As profits still dropping and interest rates increasing, agents become over-indebted and insolvable. Unable to be flowed back, panic runs through banks and the financial system collapse.

Empirically, Minsky's theory offers a basic explanation frame to East Asian crisis. In 1997, in East Asia, profits increase has lead to future profits expectations which gave rise to speculative investment and debt. The higher were speculative investment and debt, the more unstable were the financial system (Wolfson, 2002). Also, the recent financial crisis (2007) supported Minsky's hypothesis (Eggertsson and Krugman, 2010) while some other economists described it as the "*Minsky moment*" (Wray, 2008; Davidson, 2008).

However, the financial instability hypothesis is subject to some limitations. First, Minsky considered investment fuelled by credit only; it did not take into account the self-financing investment. Also, Minsky did not distinguish agents involved in the crisis and then informational asymmetry as possible strategy for companies' owners to make pessimism and panic cleared up. While the financial instability hypothesis is underpinned by a microeconomic model of the financial management of a business, it does not recognize the differences between the company's interest and those of its owners, the conflicts caused the likelihood of informational asymmetry and its implications for accounting information (Barnes, 2007). For Barnes, this limitation in the theory weakens its capacity to explain and forecast financial crises.

Beyond the above limitations revealed on Fisher's and Minsky's theories, there are still some other critics necessary to be brought up. They concern the nature of agents involved in the over-indebtedness.

4.3. FURTHER CRITICS ON FISHER'S AND MINSKY'S THEORIES

The mainstream financial instability theories agree that reimbursement defaults on debts are the major cause of bank crises. But many of them remain less explicit on the nature of economic agents merely responsible, not necessarily as guilty but as the origin (in terms of position) of the collapse chain. Fisher (1933) and Minsky (1975, 1977, 1982), in their theories, seem limited to firms only, their search of high profits. Firms' global objective is profit maximization through goods and services production activities for consumers. Even one can be claimed from classical or Keynesian thoughts, the demand expressed by consumers determines, at least in the long term, the level of production and by consequence the real profit level (as quantity of goods and services sold). Consumers and producers are then linked each one to other. Considering this type of relation, the returns on firms' investment would not depend merely on interest rates, deflation or fall in capital assets price but also, if not predominantly, on aggregate demand level expressed by consumers. Furthermore, deflation or fall in capital assets price can be seen as a consequence of aggregate demand stagnation or shortfall comparing to aggregate supply. It then comes out to consider consumers, aggregate demand providers, as the ultimate link when going back to the debt chain.
In the debt-deflation theory as well as in the financial instability hypothesis, profits decrease is pinned down as the main source of panic or pessimism through the economy leading to a negative expectations, deflation and finally to depression. Could consumers keep aggregate demand upward, it may likely counterbalance profits decrease due to interests rates rise. But as aggregate demand is supported by loans, no social but maximum profit intended, it drops drastically at some point of time, no more and no longer sustainable because of still rising inequalities and debt accumulation. This double aspect of final consumer as aggregate demand engine and at the same time its vulnerability due to income inequalities rising appears like weaknesses in Fisher's and Minsky's theories. Over-indebtedness really originates from consumers confronted with rising income inequalities and the social demonstration effect and then leads firms up to debt as well. In fact, the knife edge sustainability of aggregate demand gives an impression of positive anticipations which, really frail, encourage firms to raise their capacity of production and speculation by getting into debt. For remind as already mentioned above, Fitoussi and Saraceno (2010) stated that "the search for high-return investment by those who benefited from the increase in inequalities led to the emergence of bubbles. Net wealth became overvalued, and high asset prices gave the false impression that high levels of debt were sustainable". In other words, income inequalities would be the start-up point of debt accumulation, from households to firms, leading further to some final point which may be the financial crisis.

While Minsky contributed to push back the limits of financial instability source beyond the assumption of initial over-indebtedness of Fisher, introducing income inequalities in these models can help pushing them back farther.

5. INCOME INEQUALITY AND FINANCIAL INSTABILITY: MECHANISMS AND THEORETICAL MODEL

5.1. BASIC MECHANISMS

Let's consider a closed economy with three agents notably households (or workers), firms (or entrepreneurs) and rentiers (or capitalists) assumed to play financial institutions role.

i. **Households and firms**. Relations between households and firms occurred on two markets, labor market and goods and services market. Households offer their labor force

to fuel production activities of firms which pay back wages earned by households to consume goods and services produced by firms. According to these relations, there exist two demand regimes due to ambivalent effects of real wage on demand (Charpe, 2009). A demand regime in the Keynesian thought pushed up by wages and the other one pushed up by profits (with recessive effect of wages) corresponds to the Classical thought. Whatever the thought considered the two regimes present the common point of aggregate demand influenced by income distribution. Real wages tend to increase households' consumption as they reduce at the same time profits. And yet, in full capacity of production, firms' decision to respond to aggregate demand rise depends on their profits level. Real wages and profits indefinitely interact and determine aggregate demand level. Then in case of income inequalities partly due to real wages stagnation and profits rise, aggregate demand would diminish, everything being equal, inducing social and economic disequilibrium, leading further to profits decrease.

ii. **Households and rentiers**. Loans market describes relations between households and rentiers. In a context of income inequalities and bank deregulation, households do not have any other recourse than loans as they wish either to maintain their standards of living or to keep them upward. The higher are income inequalities, the more important is the part of consumption not covered by households' current incomes, also like their liabilities towards rentiers. Loans market is adjusted by interest rate which is rentiers' remuneration from loans activities. And the value of households' debt is directly influenced by interest rate fluctuation levels. Thus households' debt variation depends on income inequalities and interest rate. If interest rate increases, households' wealth decrease in favor of rentiers and gives rise to overall income inequalities. Debt and income inequality interact each one on other.

iii. **Firms and rentiers**. Loans market relates firms and rentiers as well. Firms' investment demand depends on the expected difference between profits and interest costs (Taylor and O'Connell, 1985). This investment is provided through self-financing and loans. Interest costs on loans go to Rentiers such as share dividends which represent the first part of net profits of firms. Self-financing or firms' internal savings is the other part of firms' net profits. Net profits are gross profits from which interest costs are retired. Share dividends and interest costs on households and firms debt represent rentiers total incomes. Rentiers spend a part of their incomes in consumption and use the other part to provide loans to the need of households and firms.

5.2. THEORETICAL MODEL

The theoretical model developed in this paper is a Keynes-Goodwin-Minsky macrodynamic-based model with a predator-prey struggle background. It's a combination of Keynes-Goodwin model and Minsky's financial instability hypothesis based model. Apart from focusing on macro-financial instability, the two models are all Keynes-underpinned and follow Lotka-Volterra predator-prey struggle model spirit. The model considers directly income inequality measured by the Gini coefficient as a dynamic state variable and it takes simultaneously into account households' debt and firms' debt without simplification assumptions. Households' debt dynamics is based on Keynes-Goodwin model while firms' debt relies on Minsky's. As mentioned above, the model involves three agents and economy is considered to be at steady state.

5.2.1 Dynamics of households' debt and income inequality

Let's consider Keynes-Goodwin model basic equations following Charpe (2009). The system is a predator-prey dynamic-based with respect to households' debt and their wage share.

$$Y_w = vY - rD_w \quad \text{with} \quad v = \frac{wL^d}{Y} = \frac{w}{z} \tag{1}$$

$$C_w = Y_w + D_w = c_w Y_w \quad \text{with} \quad c_w = c_d + c_{nd} > 1 \tag{2}$$

$$\dot{D}_w = (c_w - 1)(vY - rD_w) \tag{3}$$

$Y, w, v, L^d, z = \frac{Y}{L^d}, r$ are respectively total income, wage, wage share in total income, employment level, labor productivity and interest rate. c_w is households' marginal propensity to consume and corresponds to the sum of marginal propensity to consume durables goods (c_d) and non durable goods (c_{nd}). $\dot{D}_w$ represents households' debt variation.

Households total income (Y_w) is composed of aggregate wage from which interest costs on debt D_w are deducted. Their consumption (C_w) is financed by wage net of interest costs and debt D_w contracted so as to keep up the standards of living.

Now in this basic system, let's introduce income inequality measure directly as a state variable.

As we assumed that households' overconsumption depends on the level of income inequality, it's then possible to write down consumption as a function of income inequality. Let's consider the following theoretical argument steps:

a. *The tendency of consumption inequality to be inelastic to income inequality: income inequality do not lead (a priori) to proportionate consumption inequality (Kruger and Perri, 2005)*
b. *In case of income inequality rise, thus the fall of relative purchasing power, marginal propensity to consume will increase proportionately (Blinder, 1975)*
c. *Income inequality and consumption are closely linked through the modulation of marginal propensity to consume (Blinder, 1975)*

As a result, income inequality influences directly marginal propensity to consume (MPC). In case of income inequality rise, MPC increases as well while it decreases in the reverse case. In consequence we can write down MPC as an increasing function of income inequality like this:

$$c_w = f(IR) = c_0 + IR^{\alpha} \quad , \quad 0 < c_0 < 1 \tag{4}$$

IR is a measure of income inequality comprised between 0 and 1. α is MPC's degree of sensibility to income inequality variation. c_0 is a fix marginal propensity independent of income inequality variations. According to α, two hypotheses can be considered:

i. $If\ 0 < \alpha < 1 \longrightarrow f'(IR) > 0 \ \ and \ \ f''(IR) < 0$

In this case, the variation of MPC is less important than income inequality variation. MPC's elasticity to income inequality is less than unity. Blinder (1975) has showed in the case of the USA that for a 10% increases of income inequality, MPC increases too but less than 10%. Moreover the increase of MPC is more important for lower income than higher

income. However, Blinder's empirical research was based on the period of 1947-1972, which matched with the bank regulation period and lower income inequality level. As Blinder did not consider the periods of higher income inequality level and/or bank deregulation, it seems prudent to consider a second hypothesis.

ii. $If\ \alpha > 1 \longrightarrow f'(IR) > 0\ \ and\ \ f''(IR) > 0$

In this case, MPC varies higher than income inequality variation which induced it. MPC's elasticity to income inequality is greater than unity.

Putting equation (4) into (3) yields: $\dot{D}_w = (c_0 + IR^{\alpha} - 1)(vY - rD_w)$ (5a)

Considering in equation (5a) the derivation of $d_w = \frac{D_w}{Y}$ with respect to time, that is $\dot{d}_w = \frac{\dot{D}_w}{Y} - \hat{Y}d_w$, yields the dynamics of households debt including directly income inequality.

$$\dot{d}_w = (IR^{\alpha} + c_0 - 1)v - r(IR^{\alpha} + c_0 - 1 + \frac{\hat{Y}}{r})d_w \quad (5b)$$

The wage share v as written in equation (1) yields the following:

$$v = \frac{w}{z} \rightarrow \quad \hat{v} = \hat{w} - \hat{z} \quad \rightarrow \quad \dot{v} = (\hat{w} - \hat{z})v \quad (6)$$

We consider Phillips' curve equation as follows:

$$\hat{w} = \varphi U^{-\varepsilon} - \theta \qquad \text{with} \quad \varphi > 0 \text{ and } \varepsilon > 0 \quad (6a)$$

With the transformed and linearized Phillips' curve following Weber (2005), equation (6a) can be written like this: $\hat{w} = \varphi E - \theta$ (6b)

U, E are respectively unemployment and employment rates. φ, θ are positive constants. Considering that employment rate E variation depends on the level of capacity of production, everything being equal, E can be written, dependent of full capacity of production level corresponding to $Y^*\ and\ E^*$.

$$E = E^* + \delta\left(\frac{Y}{Y^*} - 1\right) \qquad \text{with} \quad \delta > 0 \quad (7)$$

Having $Y = Y^d = C + I$ with Y^d as aggregate demand, we can write the following

$$Y^d = Y_w + D_w + I = vY + (1 - r)D_w + I \quad (8)$$

Rewriting E incorporating equation (8) in (7), we have:

$$E = E^* + \delta\left[v + (1 - r)d_w + \frac{I}{Y^*} - 1\right] \quad (9)$$

Replace E in equation (6b) with its expression in equation (9) and then the new expression of $\hat{W}$ in equation (6) yields wage share dynamics:

$$\dot{v} = \left\{\varphi E^* + \varphi\delta\left[v + (1 - r)d_w + \frac{I}{Y^*} - 1\right] - (\theta + a)\right\}v \quad (10)$$

By introducing directly income inequality in Keynes-Goodwin basic model, we have the first two-dimensional dynamic system with respect to households' debt (as prey) and wage share (as predator)[3]:

$$\begin{cases} \dot{v} = \left\{\varphi E^* + \varphi\delta\left[v + (1-r)d_w + \frac{I}{Y^*} - 1\right] - (\theta + a)\right\}v \\ \dot{d}_w = (IR^{\alpha} + c_0 - 1)v - r(IR^{\alpha} + c_0 - 1 + \frac{\hat{Y}}{r})d_w \end{cases} \quad \textbf{(I)}$$

Now let's consider the Gini coefficient as income inequality measure. Gini coefficient basic formula can be written as follows:

$$G = \frac{2\sum_{i=1}^{n} iY_i}{n\sum_{i=1}^{n} Y_i} - \frac{n+1}{n} \quad \text{with} \quad 0 \le G \le 1$$

n = the number of individuals, Y_i individual i income such that $Y_i < Y_{i+1}$

Let's consider that there exists in the economy two groups of individuals $n_w \; and \; n_c$ with respective income such that $Y_w < Y_c \; with \; Y = Y_w + Y_c$. If we assume a priori $n = n_w + n_c = 2$ considering two representative individuals, one of each group, then as result we'll have:

$$G = \frac{1}{2}\left(\frac{Y_c - Y_w}{Y_c + Y_w}\right)$$

Furthermore, relaxing assumption that = 2 , we can consider a multiplier μ ($0 < \mu < 1$) such that when $n_w > n_c$ then μ >> 1/2 and if $n_w < n_c$ then μ << 1/2. It seems better to rewrite G as follows:

$$G = \mu(y_c - y_w) \qquad (11)$$

Replacing y_c by its following expression $y_c = 1 - y_w$ and y_w by its expression from equation (1) yields:

$$G = \mu(1 - 2v + 2rd_w) \qquad (12)$$

The derivation of G with respect to time yields:

$$\dot{G} = 2\mu\left[r\dot{d}_w - \dot{v}\right] \qquad (13)$$

Adding equation (13) to the system **(I)** yields a quite different system as follows:

---

[3] $\frac{\partial(\dot{d_w})}{\partial v} = IR^{\alpha} + c_0 - 1$. If v is higher, IR^{α} decreases and $IR^{\alpha} + c_0 - 1$ tends to be negative. In the other case, i.e v is small, IR^{α} increases and $IR^{\alpha} + c_0 - 1$ tends to be positive. As predator population rises (decreases), prey population decreases (increases). There is a dynamic between wage share variation and income inequality. This, once again, shows the necessity to consider the dynamic of income inequality.

$$\begin{cases} \dot{G} = 2\mu\left[r\dot{d}_w - \dot{v}\right] \\ \begin{cases} \dot{v} = \left\{\varphi E^* + \varphi\delta\left[v + (1-r)d_w + \frac{I}{Y^*} - 1\right] - (\theta + a)\right\}v \\ \dot{d}_w = (G^\alpha + c_0 - 1)v - r(G^\alpha + c_0 - 1 + \frac{\hat{Y}}{r})d_w \end{cases} \end{cases} \quad \textbf{(I')}$$

Now, all we need is to replace $\dot{d}_w \ et \ \dot{v}$ by their respective expressions in G dynamic equation and considering a system composed of $\dot{G}$ and one of them according to the objective interest. In the case of this paper, we consider a system of $\dot{d}_w \ and \ \dot{G}$, that is the dynamics of households' debt ratio and income inequality.

$$\begin{cases} \dot{G} = 2\mu\left[r\dot{d}_w - \dot{v}\right] \\ \dot{d}_w = (G^\alpha + c_0 - 1)v - r(G^\alpha + c_0 - 1 + \frac{\hat{Y}}{r})d_w \end{cases} \quad \textbf{(II)}$$

With $\dot{v} = \left\{\varphi E^* + \varphi\delta\left[v + (1-r)d_w + \frac{I}{Y^*} - 1\right] - (\theta + a)\right\}v$

Stability

In order to investigate the dynamic behavior of system (II), even though it is a first part of our final global dynamic system, let's consider its Jacobian matrix and characteristic equation as follows:

The Jacobian matrix J gives:

$$J = \begin{bmatrix} 2\mu r\alpha G^{\alpha-1}(v - rd_w) & -2\mu[r^2\left(G^\alpha + c_0 - 1 + \frac{\hat{Y}}{r}\right) + \varphi\delta v(1-r)] \\ \alpha G^{\alpha-1}(v - rd_w) & -r(G^\alpha + c_0 - 1 + \frac{\hat{Y}}{r}) \end{bmatrix}$$

And the characteristic equation of J is given by: $|\lambda I - J| = \lambda^2 - (trace\ J)\lambda + |J| = 0$

System (II) is real and stable if and only if $|J| > 0$ and $(trace\ J) < 0$

$|J| = 2\mu\alpha G^{\alpha-1}\varphi\delta v(1-r) > 0$. In consequence, system (II) has an equilibrium point economically possible, $(G^*, d_w^*) > (0,0)$. If $(trace\ J) < 0$ then the system is stable at the vicinity of the equilibrium point(G^*, d_w^*).

$$trace\ J = 2\mu r\alpha G^{\alpha-1}(v - rd_w) - r(G^\alpha + c_0 - 1 + \frac{\hat{Y}}{r})$$

$$trace\ J < 0^4 \quad \longrightarrow \quad G^\alpha[\alpha 2\mu G^{-1}(v - rd_w) - 1] < c_0 - 1 + {\hat{Y}}/{r}$$

[4]If $2\mu r\alpha G^{\alpha-1}(v - rd_w) < r(G^\alpha + c_0 - 1 + \frac{\hat{Y}}{r})$ then the characteristic equation has two roots with negative real parts (local stability). In the case of *trace J = 0*, the equation has a pair of pure imaginary roots. In case that *trace J* is positive the real part of both roots become positive (local instability).

We assume that there exists $\alpha_0 > 0$ such that $G^{\alpha_0}[\alpha_0 2\mu G^{-1}(v - rd_w) - 1] = c_0 - 1 + {}^{\hat{Y}}/_{r}$

and that if $0 < \alpha < \alpha_0$ then $G^{\alpha}[\alpha 2\mu G^{-1}(v - rd_w) - 1] < c_0 - 1 + {}^{\hat{Y}}/_{r}$

According to the value of the right term, two cases can be considered.

1st case: $\hat{Y} > 0$, ***r is lower,*** ${}^{\hat{Y}}/_{r} > 0$ ***and*** α_0 ***is higher***.

In this case the system's stability interval $]0; \alpha_0[$ is large. α_0 (α) is higher and much more as $\hat{Y}$ is higher or r is lower. The higher is α, the more sensible is the MPC to income inequality leading to a higher propensity for households to be indebted. This case corresponds to a situation of high households' debt ratio variation which matches with high economic growth and low interest rate. It's a situation of economic boom.

2nd case: $\hat{Y} \leq 0$, ${}^{\hat{Y}}/_{r} \leq 0$ ***and*** α_0 ***is lower***

In this case, interval$]0; \alpha_0[$ is narrow. α_0 (α) is lower, less higher than the first case. MPC is less sensible to income inequality variations leading to a lower propensity of households to be indebted. This second case corresponds to a situation of low households' debt ratio variation which matches with low or negative economic growth and high interest rate. It's a situation of economic recession and depression.

Whatever the above two cases, the system is locally stable when $0 < \alpha < \alpha_0$. In the case of $\alpha > \alpha_0$, the system is locally totally instable. Hopf bifurcation occurs when $\alpha = \alpha_0$.
In order to appreciate the dynamic behavior of system (II), we use phase diagram[5] as out-lined below. There are two cases according to the value of α as MPC's elasticity to income inequality.

Case 1: $\alpha < 1$

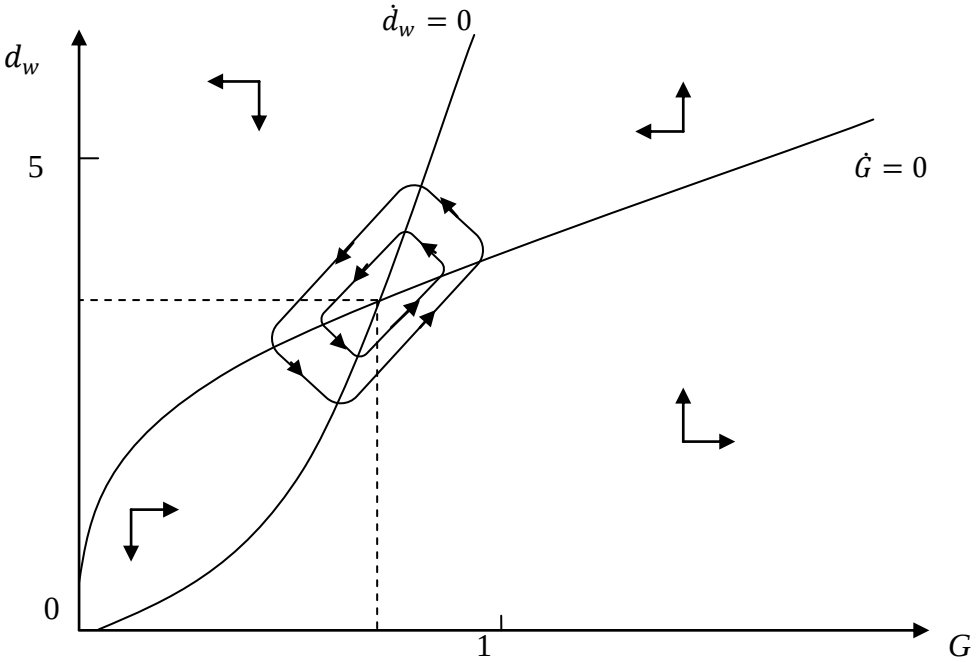


Fig. 3a: System stability at the vicinity of equilibrium point.

[5] See the phase lines equations in the appendix C.

***Case 2* : α > 1**

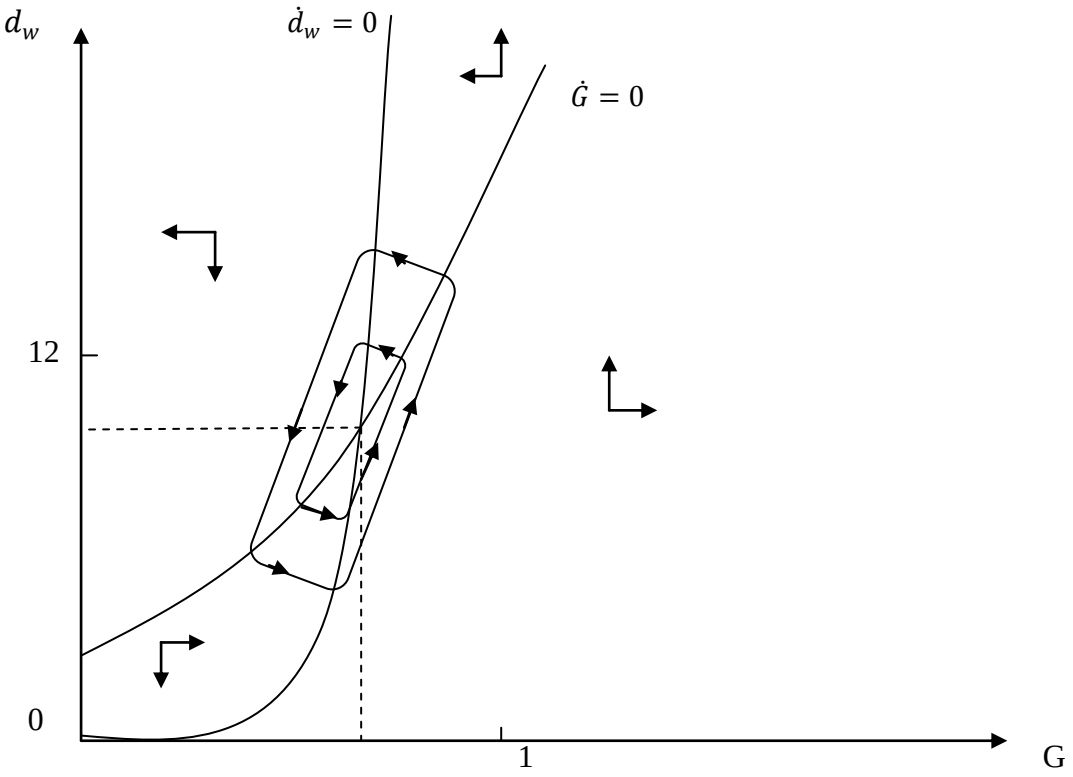


Fig. 3b: system stability at the vicinity of equilibrium point

As MPC is very elastic to income inequality (α > 1), much higher is the debt ratio variation.

5.2.2 Dynamics of firms' debt and aggregate demand

Let's consider two groups of basic equations for rentiers and firms. For rentiers, we have:

$$Y_c = (1 - s_F)(\Pi - rD_F) + \rho V + rD \quad (14)$$

$$D = D_w + D_F \quad (15)$$

$$V = \gamma D \quad \text{with} \quad \gamma > 0 \quad (16)$$

$$S_c = s_c[(1 - s_F)(\Pi - rD_F) + \rho V + rD] \quad \text{with } s_c > 0 \quad (17)$$

Rentiers' current income (Y_c) consists of three parts, i.e., the dividend ($(1 - s_F)(\Pi - rD_F)$), the interest receipt on debt (rD) and interest receipt on safe assetρV. For simplicity, safe asset V is assumed to be proportional to the debt and is remunerated by the market rate of interest ρ. Rentiers save a part of their income which is S_c.

Now for firms we have:

$$\Pi = Y - wL^d = (1 - v)Y \quad (18)$$

$$\Pi_{net} = \Pi - rD_F = s_F(\Pi - rD_F) + (1 - s_F)(\Pi - rD_F) \quad (19)$$

Where Π, Y, D_F are respectively the gross profit, the output level and firms' debt. The net profit (gross profit net of interest costs) is distributed in two parts. A first part s_F $(0 < s_F < 1)$consists of the intern retention rate as firms' self-financing. A second part $(1 - s_F)$, the dividend, goes to capitalists (rentiers). Firms get into debt to finance the other part of their investments not covered by the self-financing. Here, it seems to focus on two important points. On the first hand, it is well established that firms' debt rate depends on investment level. On the second hand, according to Minsky, firms' investment decision relies on future profits to be generated by the investment. But as mentioned above, profit also depends on aggregate demand level expressed by consumers or at least to be caught. In this case, firms' investment decision is taken so as to catch a demand surplus, source of future profits, by increasing the aggregate supply then their capacity of production. In consequence a relation between firms' debt and the aggregate demand can be laid down.

The dynamics with respect to firms' debt and aggregate demand represent the other part of our global final system. For determining these dynamics, we entirely follow Asada (2001). The dynamic model of Minsky cycle developed by Asada is a predator-prey struggle based model of debt and capital. Asada considers the microeconomic foundation of the investment function with a debt effect and started from the criterion of optimal investment of Kalecki (1937) which is an alternative to Keyne's (1936) famous criterion. Kalecki (1936) stipulated that the marginal efficiency of investment (m) is the sum of the market rate of interest (ρ) and the marginal risk (σ): $m = \rho + \sigma$ (20a)

Considering the Kaleckian principle of increasing risk, the marginal risk is an increasing function of the rate of investment (or capital accumulation rate g) and some other variables such as rate of profit, rate of interest, debt-capital ratio, etc. Then Asada wrote down the marginal efficiency of investment function as follows:

$$m(g;\pi) = \rho + \sigma(g;\pi,\rho,\bar{s}_F,d_F) \quad (20b)$$

With $m(g;\pi) = {}^{\pi}/_{\emptyset_1'(g)}$ and $\sigma(g;\pi,\rho,\bar{s}_F,d_F) = \emptyset_2'(\cdot)$

$\pi = \frac{\Pi}{K}, d_F = \frac{D_F}{K}$ are respectively the rate of profit and firms' debt-capital ratio. K corresponds to the capital level

The investment function with debt effect is obtained by solving equation (20b) with respect to g as follows:

$$g = g(\pi,\rho,\bar{s}_F,d_F) \quad (21)$$

$\emptyset_1$ captures the hypothesis of the increasing adjustment cost of investment introduced by Uzawa (1969) as Asada (2001) precised while $\emptyset_2$captures the "Kalecki effect" of marginal risk. Firms' debt varies with respect to time following the relationship below:

$$\dot{D}_F = \emptyset_1(g)K - s_F(\pi K - rD_F) \quad (22)$$

where $r = r(\rho,d) = \rho + \tau(d)$and $r > \rho$

Replacing the derivation with respect to time of firms' debt-capital ratio, i.e. $\dot{d}_F = \frac{\dot{D}_F}{K} - d_F g$, in equation (22) we have the dynamics of debt-capital ratio:

$$\dot{d}_F = \emptyset_1\big(g(\pi, \rho, s_F, d_F)\big) - s_F[\pi - r(\rho, d)d_F] - g(\pi, \rho, s_F, d_F)d_F \quad (23)$$

We still follow Asada (2001) for the dynamics of aggregate demand who considered the Keynesian quantity adjustment process in the goods market.

$$\dot{y} = \alpha'(c + h - y), \quad \alpha' > 0 \quad (24)$$

$y = {}^{Y}/_{K}, c = {}^{C}/_{K}, C = C_w + C_c, h = {}^{I'}/_{K} = \emptyset_1(g)$ are respectively the output-capital ratio, the consumption expenditures capital ratio, the consumption expenditures consist of households consumption and capitalists consumption, the ratio of investment expenditure including adjustment cost over capital.

In fact, Asada (2001) did not take into account households' debt ratio and then considered their income as equal to their wage and exactly equal to their consumption expenditure. In contrast in this paper where households' excessive debt is considered as the upstream problem of the global system indebtedness, they spend in consumption their wage income net of interest costs on their ongoing liabilities. Replacing in equation (2) Y_w by its expression of equation (1), we can write households' and rentiers' consumption expenditure with their capital ratio as follows:

$$C_w = vY + (1 - r)D_w$$

$$\frac{C_w}{K} = vy + (1 - r)d_w y$$

$$C_c = Y + (1 - r)D_w + (1 - s_c)[\rho V + r(\rho, d)D] - (1 - v)[s_F + (1 - s_F)s_c]Y$$

$$\frac{C_c}{K} = y + (1 - r)d_w y + (1 - s_c)[\rho\gamma + r(\rho, d)](d_F + d_w y) - (1 - v)[s_F + (1 - s_F)s_c]y$$

Then the output capital ratio dynamics gives:

$$\dot{y} = \alpha'\left\{\emptyset_1\left(g\big((1 - v)y, \rho, s_F, d_F\big)\right) + (1 - s_C)[\rho\gamma + r(\rho, d)][d_F + (1 - r)d_w y] - (1 - v)s_F + (1 - s_F)s_C y\right. \quad (25)$$

As given above aggregate demand $y^d = y$ (Asada, 2006). Asada's (2001) dynamic system[6] in which we introduce households' debt can be written as follows:

$$\begin{cases} \dot{d}_F = \emptyset_1\left(g\big((1 - v)y^d, \rho, s_F, d_F\big)\right) - s_F[(1 - v)y^d - r(\rho, d)d_F] - \\ \qquad g((1 - v)y^d, \rho, s_F, d_F)d_F \\ \dot{y}^d = \alpha'\left\{\emptyset_1\left(g\big((1 - v)y^d, \rho, s_F, d_F\big)\right) + (1 - s_C)[\rho\gamma + r(\rho, d)][d_F + (1 - r)d_w y^d] - \right. \\ \qquad\qquad\qquad\qquad (1 - v)[s_F + (1 - s_F)s_C]y^d \end{cases} \quad \textbf{(III)}$$

[6] For more details on the model, the local stability proofs, the stability hypotheses and the diagram phase representation, see Asada T. "Nonlinear Dynamics of Debt and Capital: A Post-Keynesian Analysis" (2001).

5.2.3 The Keynes-Goodwin-Minsky Model

The second originality of the model we are developing in this paper consists of considering simultaneously dynamics of households' and firms' debt ratios. All we need is to bring together systems (II) and (III).
As the output-capital ratio is $y = {}^{Y}/_{K}$, we have the following

$$\hat{y} = \hat{Y} - \hat{K} = \hat{Y} - g \quad \longrightarrow \quad \hat{Y} = \hat{y} + g = \hat{Y}^d = \hat{y}^d + g$$

As a result, households' debt ratio dynamics of equation (5b) can be rewritten like this:

$$\dot{d}_w = (G^\alpha + c_0 - 1)v - \left[r(G^\alpha + c_0 - 1) + g\Big((1-v)y^d, \rho, s_F, d_F\Big)\right] d_w - d_w \hat{y}^d \qquad (26)$$

Considering systems (II) and (III) with respect to equation (26) yields our final dynamic system (which we can note (**IV**)), the Keynes-Goodwin-Minsky model as follows:

$$\begin{cases} \dot{G} = 2\mu\left[r\dot{d}_w - \dot{v}\right] & \textbf{(\textit{i})} \\ \dot{d}_w = (G^\alpha + c_0 - 1)v - \left[r(G^\alpha + c_0 - 1) + g\Big((1-v)y^d, \rho, s_F, d_F\Big)\right] d_w - d_w \hat{y}^d & \textbf{(\textit{ii})} \\ \dot{d}_F = \emptyset_1\left(g\Big((1-v)y^d, \rho, s_F, d_F\Big)\right) - s_F[(1-v)y^d - r(\rho, d)d_F] - g((1-v)y^d, \rho, s_F, d_F)d_F & \textbf{(\textit{iii})} \\ \dot{y}^d = \alpha' \left\{ \begin{matrix} \emptyset_1\left(g\Big((1-v)y^d, \rho, s_F, d_F\Big)\right) + (1-s_C)[\rho\gamma + r(\rho, d)][d_F + (1-r)d_w y^d] - \\ (1-v)[s_F + (1-s_F)s_C]y^d \end{matrix} \right\} & \textbf{(\textit{iv})} \end{cases}$$

6. STABILITY OF THE MODEL

For simplicity, we consider the following abbreviations:

$$(i) = f_1(G, d_w, d_F, y^d) \quad ; \quad (ii) = f_2(G, d_w, d_F, y^d)$$
$$(iii) = f_3(G, d_w, d_F, y^d) \quad ; \quad (vi) = f_4(G, d_w, d_F, y^d)$$

The jacobian matrix[7] J' (see first derivatives' expressions in *Appendix A*) and its characteristic equation are as follows:

[7] For example f_{11} represents the first derivation of function f_1 with respect to the first variable, i.e. G. f_{23} represents the first derivation of function f_2 with respect to the third variable, i.e. d_F . And so on.

$$J' = \begin{pmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{pmatrix}$$

$|\lambda I - J'| = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4$

$a_1 = -(trace\, J')$, $a_2 = sum\ of\ all\ principal\ second - minors\ of\ J'$
$a_3 = -(sum\ of\ all\ principal\ third - minors\ of\ J')$ and $a_4 = \det(J')$.
(See *Appendix B* for coefficients a_i expressions).

For stability criterion, we use Routh-Hurwitz conditions for stable roots. These conditions stipulate that a dynamic system (four-dimensional system in this case) is locally stable if and only if: $a_i > 0$ $(i = 1, 2, 3, 4)$ and $\Phi = a_1a_2a_3 - a_1^2a_4 - a_3^2 > 0$ with respect to the characteristic equation.

***First conditions*:** $\boldsymbol{a_i > 0}$ $\boldsymbol{(i = 1, 2, 3, 4)}$
$a_1 = -(f_{11} + f_{22} + f_{33} + f_{44})$

Given the stability conditions of system (II), $f_{11} > 0$, $f_{22} < 0$ $with$ $|f_{11}| < |f_{22}|$

Also, given the stability conditions of system (III) (Asada, 2001), $(f_{44}f_{33} - f_{43}f_{34}) > 0$, $f_{44} > 0$, $f_{33} < 0$ $with$ $|f_{44}| < |f_{33}|$ in the case that $\emptyset_1'(g), g_\pi$ and $|g_{d_F}|$ are sufficiently large at the equilibrium point and $\lim_{s_c \to 1}(f_{33}f_{44} - f_{34}f_{43}) > 0$.

In consequence, $trace\, J' < 0$ $and\ then$ $a_1 > 0$
Moreover, we prove by simple calculation and without any other assumption that a_2 , a_3 and a_4 are positive (See *Appendix B*).

***Second conditions*:** $\boldsymbol{\Phi = a_1a_2a_3 - a_1^2a_4 - a_3^2 > 0}$ *(see* $\boldsymbol{\Phi}$ *expression in Appendix B)*

Assumption 1*:* $|J_{24}'| = f_{22}f_{44} - f_{24}f_{42} > 0$

$f_{22} = -r(G^\alpha + c_0 - 1) - y^d - g < 0$
$f_{24} = -g_{y^d}d_w < 0$
$f_{42} = \alpha'\{(1 - s_c)[\rho\gamma + r(\rho, d)](1 - r)y^d\} > 0$
$f_{44} = \alpha'(1 - v)\{\emptyset_1'(g)g_\pi - [s_F + (1 - s_F)s_c]\} + \alpha'(1 - s_c)(\rho\gamma + r)(1 - r)d_w > 0$

The inequality in assumption 1 will be satisfied if $|f_{24}|$, i.e. g_{y^d}, is sufficiently large. Furthermore, we consider $\lim_{s_c \to 1}(f_{22}f_{44} - f_{24}f_{42})$where $\lim_{s_c \to 1}(f_{24}f_{42}) = 0$ $(f_{42} \to 0)$.

Then $\lim_{s_c \to 1}(f_{22}f_{44} - f_{24}f_{42}) = \lim_{s_c \to 1}(f_{22}f_{44})$ with $f_{22} = -r(G^{\alpha} + c_0 - 1) - y^d - g$ and $f_{44} = \alpha'(1-v)\{\emptyset_1'(g)g_{\pi} - 1\}$.
f_{22} expression is unchanged while the positive part of f_{44} equals 0. The inequality in the assumption will be satisfied if $f_{44} < 0$. One can easily noticed that the inequality $f_{44} < 0$ is for the more possible.
In these conditions, we can argued that at the equilibrium point when $s_c \to 1$, $|J_{24}'| > 0$

The satisfaction of the assumption's inequality yields two results.

First, if $f_{44} < 0$ *and* $|J_{24}'| > 0$, $f_{22} + f_{44} = trace\, J_{24}' < 0$, thus the equilibrium of the system composed of equations (*ii*) and (*iv*), i.e. households' debt ratio and aggregate demand, is locally stable.
Second, as $\left|J_{24}'\right| > 0$, $\Phi = a_1a_2a_3 - a_1^2a_4 - a_3^2 > 0$, and then the system (IV) is locally stable.

The dynamic behavior of system (IV) is represented below on figure 4[8].

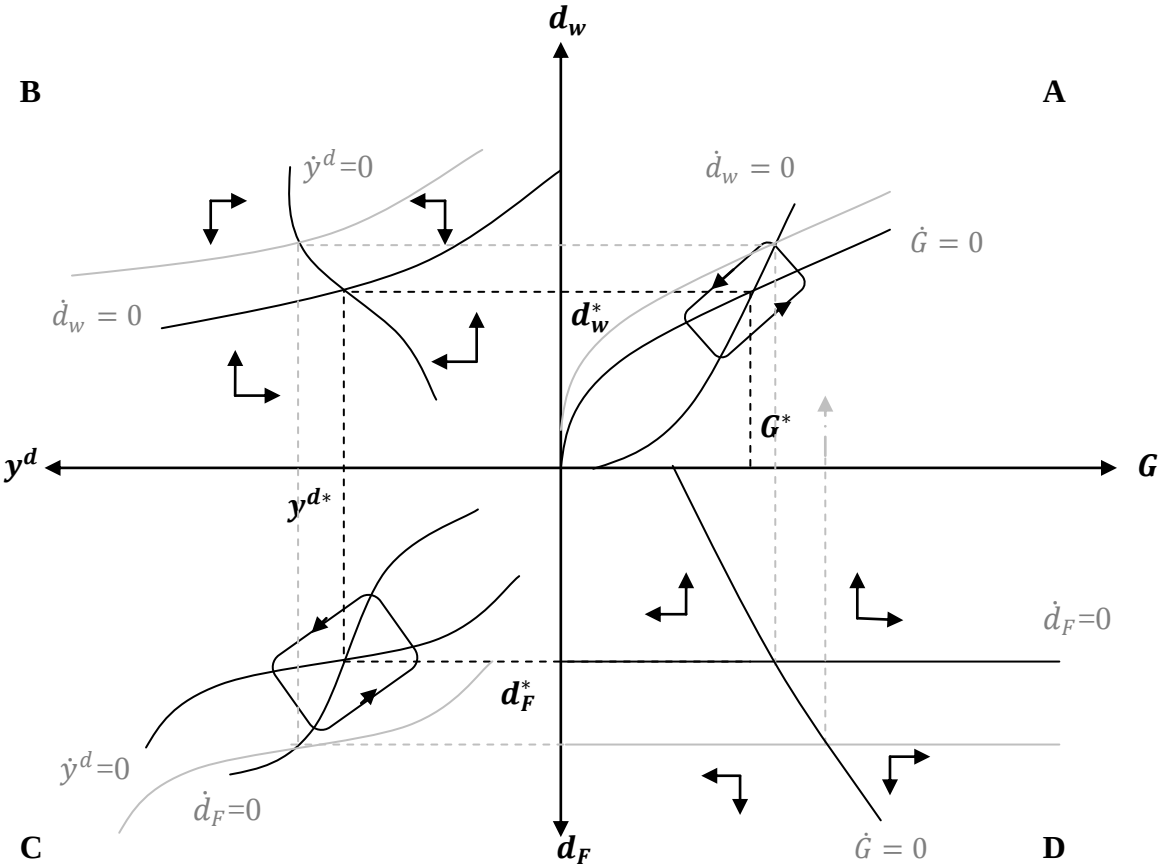


Fig. 4: Phase diagram of system (IV) in the case where $\alpha < 1$

[8] For the expression of phase line of equations *(ii)* and *(iv)*, see Appendix C. For the expression of phase line of equations *(i)* and *(iii)*, see Appendix C.

7. ECONOMIC INTERPRETATIONS

The dynamics of system (IV), as represented above, have four parts A, B, C and D.

Part **A** shows income inequality impact on households' debt ratio which cumulative level depends on MPC's sensibility to income inequality. The higher this sensibility is (α high), the faster grows households' debt ratio. In face of high income inequalities, as relative purchasing power of lower incomes with higher MPC falls, effective demand falls as well. Households' behavior of keeping up/upward their standards of living by going into debt sustains effective demand. A positive variation of households' debt leads to a positive variation of aggregate demand (**B**). This mechanism is eased by financial deregulation. The leverage effect of credit on aggregate demand gives an impression of positive expectations on firms' future profits. These expectations, really frail, encourage firms to raise their capacity of production and speculation by getting into debt. With the incitation of high-returns investment and loans, firms and rentiers create an imaginary world of exceptional economic enthusiasm and consumers are submerged by new goods and services publicity hype and soft credit conditions. Social demonstration effect rises and then the conjunction of all these phenomenon leads progressively to an economic boom amplified by speculative and Ponzi-financed investments (α high with $\hat{Y} > 0$ and r low). As aggregate demand is still rising, the enthusiasm of positive expectations, at least confirmed, rises as well as firms' debt ratio (**C**). Part **D** shows the dynamics between firms' debt ratio and the Gini coefficient. When firms' debt ratio rises, income inequality rises too as consequence of precedent mechanisms. Firms' decision of debt ratio increasing is taken on the base of high future profit, at least higher than wage costs induced (g_π high). The profits are distributed to rentiers and for firms' self-financing. All these situations worsen income inequality comparing to its start-up level. At that point of time debt and income inequality act and react on each other and make a vicious circle emerged. Income inequality has induced debt ratio rise which in turn leads to overall income inequality increase.

The situation as described above from part **A** to part **D** will still ongoing. The economic boom enthusiasm now sets all economic agents on fire and gives rise further to bubbles. But while debt is accumulating interest rate begins to increase, MPC's sensibility to income inequality diminishes (α less high), as for firms (g_{d_F} negative and $\left|g_{d_F}\right|$ high) till insolvency. As capital accumulation rate reached a maximum level, its increasing adjustment cost is higher too ($\emptyset_1'(g)$ high). At insolvency limit point, households try to pay off and get rid of their debt. With interest rate increase and debt accumulation weight, households are constrained to do what they were supposed to do in the past: reduce their standards of living according to their economic standing. Now, they are less sensible to income inequality even it is reaching a top level. Aggregate demand loses the credit leverage effect and depresses drastically (α lesser with $\hat{Y} < 0, r$ higher). This stability condition of the equilibrium of equations (*i*) and (*ii*) matches with the one of equations (*ii*) and (*iv*), i.e. $\lim_{s_c \to 1}(f_{44}) < 0$ (f_{44} is the growth rate of aggregate demand thus economic growth). Positive anticipations changed to negative anticipations while interest rate is still rising making the real value of debt increased. Profit rate drops and induces a distress selling of assets. Assets prices drop drastically and lead further to overall profits fall. Unemployment increases and wage share which goes to households, falls as well reducing their reimbursement capacity. As mentioned above rentiers has engaged a maximum of their funds in

providing loans in other to benefit from the high returns ($s_c \rightarrow 1$). Reimbursement defaults lead to bank illiquidity, panic and finally to financial collapse.

The dynamic system developed in this paper as Keynes-Goodwin-Minsky model, clearly shows how income inequalities lead to financial instability through over-indebtedness confirming our hypothesis of positive causality.

8. CONCLUSION

Putting together Keynes-Goodwin model and Minsky's financial instability hypothesis, we developed a model of financial instability which considers income inequality as direct dynamic state variable and takes simultaneously into account household's debt and firm's debt ratios. As a result, the Keynes-Goodwin-Minsky model shows a positive causality between income inequality and financial instability through the channel of over-indebtedness. The model's mechanisms show that the positive anticipations were fed by a rising aggregate demand. But aggregate demand itself is supported by credit as recourse for lower incomes households – with decreasing purchasing power when facing persistent income inequalities – to maintain and/or keep upward their living standards. The aggregate demand sustainability is then thrown on a knife edge making its sustainability unsustainable. With his *financial instability hypothesis*, Minsky had gone beyond Fisher's *debt-deflation theory* (1933) by giving theoretical explanations to the over-indebtedness initial assumption. This paper has tried to go beyond Minsky by pointing out, in a theoretical manner with stability proof, the income inequality as the main cause of financial instability.

However, this model is a step towards further ones as it relies on the assumption of price rigidity and no government. Given that financial crisis often make public debt increased (Reinhart and Rogoff, 2010) so as to create a relapse, taking into account the public debt in this model will be for a serious further research issue.

Appendix A

$f_{11} = 2\mu r\alpha G^{\alpha-1}(v - rd_w) > 0$
$f_{12} = -2\mu\left[r^2\left(G^{\alpha} + c_0 - 1 + \frac{\hat{y}^d + g}{r}\right) + \varphi\delta v(1-r)\right] < 0$
$f_{13} = -2\mu r(g_{d_F} d_w) > 0 \quad , \quad f_{14} = -2\mu r d_w - g_{y^d} d_w < 0$
$g_{d_F} < 0$ (See Asada, 2001) ; $g_{y^d} > 0$ (capital accumulation rate is an increasing function of aggregate demand).

$f_{21} = \alpha G^{\alpha-1}(v - rd_w) > 0 \quad , f_{22} = -r(G^{\alpha} + c_0 - 1) - y^d - g < 0$
$f_{23} = -g_{d_F} d_w > 0 \quad f_{24} = -g_{y^d} d_w < 0$

$f_{31} = 0, \quad f_{32} = 0, \quad f_{33} = [\emptyset_1'(g) - d_F]g_{d_F} - g + s_F(r_d d_F + r) < 0$
$f_{34} = (1-v)[(\emptyset_1'(g) - d_F)g_{\pi} - s_F] > 0$
$g_{\pi} > 0$ (Capital accumulation rate is an increasing function of profit rate)
$\emptyset_1'(g) > 0 \ et \ \emptyset_1'(g) - d_F > 0$ (See Asada, 2001).

$f_{41} = 0, \quad f_{42} = \alpha'\{(1-s_c)[\rho\gamma + r(\rho, d)](1-r)y^d\} > 0$
$f_{43} = \alpha'\{\emptyset_1'(g)g_{d_F} + (1-s_c)[r_d d_F + \rho\gamma + r]\} < 0$
$f_{44} = \alpha'(1-v)\{\emptyset_1'(g)g_{\pi} - [s_F + (1-s_F)s_c]\} + \alpha'(1-s_c)(\rho\gamma + r)(1-r)d_w > 0$

Appendix B

$a_1 = -(f_{11} + f_{22} + f_{33} + f_{44})$
$f_{11} > 0 , f_{22} < 0 \ et \ |f_{11}| < |f_{22}| \quad , (f_{44}f_{33} - f_{43}f_{34}) > 0 \ , f_{44} > 0 , f_{33} < 0 \ et \ |f_{44}| < |f_{33}|$

$a_1 > 0$

$$a_2 = \begin{vmatrix} f_{33} & f_{34} \\ f_{43} & f_{44} \end{vmatrix} + \begin{vmatrix} f_{22} & f_{24} \\ f_{42} & f_{44} \end{vmatrix} + \begin{vmatrix} f_{22} & f_{23} \\ f_{32} & f_{33} \end{vmatrix} + \begin{vmatrix} f_{11} & f_{14} \\ f_{41} & f_{44} \end{vmatrix} + \begin{vmatrix} f_{11} & f_{13} \\ f_{31} & f_{33} \end{vmatrix} + \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix}$$

$a_2 = (f_{44}f_{33} - f_{43}f_{34}) + (f_{44}f_{22} - f_{42}f_{24}) + f_{22}f_{33} + f_{11}f_{44} + f_{11}f_{33} + (f_{11}f_{22} - f_{12}f_{21})$

$a_2 = (f_{44}f_{33} - f_{43}f_{34}) + (f_{44}f_{22} - f_{42}f_{24}) + f_{33}(f_{22} + f_{11}) + f_{11}f_{44} + (f_{11}f_{22} - f_{12}f_{21})$

$(f_{44}f_{33} - f_{43}f_{34}) > 0 \ et \ (f_{11}f_{22} - f_{12}f_{21}) > 0$
$f_{22} < 0, \ f_{33} < 0 \ or \ |f_{22}| > |f_{11}| \ alors \ f_{33}(f_{22} + f_{11}) > 0 \ ; f_{11}f_{44} > 0$

Whatever the sign of $(f_{44}f_{22} - f_{42}f_{24})$, a_2 would be positive and large. It would be larger if $(f_{44}f_{22} - f_{42}f_{24})$ is positive.

$$a_3 = -\begin{vmatrix} f_{12} & f_{23} & f_{24} \\ f_{32} & f_{33} & f_{34} \\ f_{42} & f_{43} & f_{44} \end{vmatrix} - \begin{vmatrix} f_{11} & f_{13} & f_{14} \\ f_{31} & f_{33} & f_{34} \\ f_{41} & f_{43} & f_{44} \end{vmatrix} - \begin{vmatrix} f_{11} & f_{12} & f_{14} \\ f_{21} & f_{22} & f_{24} \\ f_{41} & f_{42} & f_{44} \end{vmatrix} - \begin{vmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{vmatrix}$$

$a_3 = -(f_{44}f_{33} - f_{43}f_{34})(f_{11}+f_{22}) - (f_{11}f_{22} - f_{12}f_{21})(f_{44}+f_{33}) - f_{42}[(f_{21}f_{14} - f_{11}f_{24}) + f23f34-f33f24$

$f_{11}+f_{22} < 0,\ f_{44}f_{33} - f_{43}f_{34} > 0,\ f_{11}f_{22} - f_{12}f_{21} > 0,\ f_{44}+f_{33} < 0$ then,

$-(f_{44}f_{33} - f_{43}f_{34})(f_{11}+f_{22}) - (f_{11}f_{22} - f_{12}f_{21})(f_{44}+f_{33}) > 0$;

$(f_{21}f_{14} - f_{11}f_{24}) = -\alpha G^{\alpha-1}(v - rd_w)\left(2\mu r d_w + g_{y^d} d_w\right) + 2\mu r \alpha G^{\alpha-1} d_w (v - rd_w) g_{y^d}$

$(f_{21}f_{14} - f_{11}f_{24}) = \alpha d_w (v - rd_w) G^{\alpha-1}\left[2\mu r\left(g_{y^d} - 1\right) - g_{y^d}\right] < 0$

$-f_{42}[(f_{21}f_{14} - f_{11}f_{24})] > 0$;

$f_{23}f_{34} > 0, -f_{33}f_{24} < 0 \quad and \quad -f_{42}[(f_{23}f_{34} - f_{33}f_{24})] > ou\ < 0$

Whatever the sign of $-f_{42}[(f_{23}f_{34} - f_{33}f_{24})]$, the sum of the three other terms is sufficiently large so that $a_3 > 0$.

$$a_4 = \det(J') = f_{11}\begin{vmatrix} f_{22} & f_{23} & f_{24} \\ f_{32} & f_{33} & f_{34} \\ f_{42} & f_{43} & f_{44} \end{vmatrix} - f_{21}\begin{vmatrix} f_{12} & f_{13} & f_{14} \\ f_{32} & f_{33} & f_{34} \\ f_{42} & f_{43} & f_{44} \end{vmatrix} \qquad (f_{31} = f_{41} = 0)$$

$a_4 = (f_{44}f_{33} - f_{43}f_{34})(f_{11}f_{22} - f_{12}f_{21}) + f_{42}[f_{33}(f_{21}f_{14} - f_{11}f_{24}) + f_{34}(f_{11}f_{23} - f_{13}f_{21})]$

$(f_{44}f_{33} - f_{43}f_{34})(f_{11}f_{22} - f_{12}f_{21}) > 0$
$f_{33} < 0,\ (f_{21}f_{14} - f_{11}f_{24}) < 0, \quad then \quad f_{42}f_{33}(f_{21}f_{14} - f_{11}f_{24}) > 0$; But,

$f_{11}f_{23} - f_{13}f_{21} = -2\mu r \alpha G^{\alpha-1}(v - rd_w) g_{d_F} d_w + 2\mu r g_{d_F} d_w \alpha G^{\alpha-1}(v - rd_w) = 0,$

$a_4 > 0.$

$\Phi = a_1 a_2 a_3 - a_1^2 a_4 - a_3^2 = a_3(a_1 a_2 - a_3) - a_1^2 a_4$

$a_1 = -(f_{11} + f_{22} + f_{33} + f_{44})$

$a_2 = |J'_{34}| + |J'_{12}| + |J'_{24}| + f_{33}(f_{11} + f_{22}) + f_{11}f_{14}$

$a_3 = -|J'_{34}|(f_{11} + f_{22}) - |J'_{12}|(f_{33} + f_{44}) - f_{42}(f_{21}f_{14} - f_{11}f_{24})$

$$a_4 = |J_{34}^{'}||J_{12}^{'}| + f_{42}f_{33}(f_{21}f_{14} - f_{11}f_{24})$$

$$\begin{aligned}\Phi = {} & |J_{24}^{'}|(f_{11} + f_{22})^2 + |J_{24}^{'}|(f_{33} + f_{44})^2 \\ & + |J_{34}^{'}|(f_{11} + f_{22})(f_{33} + f_{44})(|J_{34}^{'}| + |J_{24}^{'}| - 2|J_{12}^{'}|) \\ & + f_{42}f_{33}(f_{11} + f_{22})(f_{21}f_{14} - f_{11}f_{24})(f_{33} + f_{44}) \\ & + f_{42}(f_{33} + f_{44})(f_{21}f_{14} - f_{11}f_{24})(-f_{11}f_{22} - f_{33}f_{44} + f_{12}f_{21}) \\ & + f_{33}|J_{12}^{'}|(f_{11} + f_{22})(f_{33} + f_{44})(f_{11} + f_{22} + f_{33} + f_{44}) \\ & - f_{42}(f_{21}f_{14} - f_{11}f_{24})[f_{33}(f_{11} + f_{22})(f_{33} + f_{44}) + f_{42}(f_{21}f_{14} - f_{11}f_{24})] \\ & + f_{33}|J_{34}^{'}|(f_{11} + f_{22})^2[f_{11} + f_{22} + f_{33} + f_{44}] \\ & + f_{42}|J_{34}^{'}|(f_{21}f_{14} - f_{11}f_{24})[(f_{33} + f_{44}) - (f_{11} + f_{22})] \\ & + f_{42}|J_{24}^{'}|(f_{33} + f_{44})(f_{21}f_{14} - f_{11}f_{24}) \\ & + |J_{12}^{'}|(|J_{24}^{'}| + |J_{12}^{'}|)(f_{11} + f_{22})(f_{33} + f_{44}) \\ & + f_{42}(f_{11} + f_{22})(f_{21}f_{14} - f_{11}f_{24})(|J_{12}^{'}| + |J_{24}^{'}|)\end{aligned}$$

Apart from the expression $-f_{42}(f_{21}f_{14} - f_{11}f_{24})[f_{33}(f_{11} + f_{22})(f_{33} + f_{44}) + f_{42}(f_{21}f_{14} - f_{11}f_{24})]$ which is negative, all the others are positive if $|J_{24}^{'}|$ is positive. Then $\Phi > 0$

Appendix C: Phase lines equations

- ***System (II)***

$$\begin{cases} d_{w,\dot{d}_w=0} = \dfrac{v[G^{\alpha} + c_0 - 1]}{r\left[G^{\alpha} + \frac{\hat{Y}}{r} + c_0 - 1\right]} \\ d_{w,\dot{G}=0} = \dfrac{rvG^{\alpha} - v\left[\varphi E^{*} + \varphi\delta\left(v + \frac{I}{Y^{*}} - 1\right) - (\theta + a) - r(c_0 - 1)\right]}{r^2G^{\alpha} + r^2\left(c_0 - 1 + \frac{\hat{Y}}{r}\right) + v\varphi\delta\,(1 - r)} \end{cases}$$

- ***System (IV): equations (ii) and (iv)***

$$\begin{cases} \dot{d}_w = (G^{\alpha} + c_0 - 1)v - \left[r(G^{\alpha} + c_0 - 1) + g\left((1 - v)y^d, \rho, s_F, d_F\right)\right]d_w - d_w\hat{y}^d \qquad (ii) \\ \dot{y}^d = \alpha^{'}\left\{\begin{matrix}\emptyset_1\left(g\left((1 - v)y^d, \rho, s_F, d_F\right)\right) + (1 - s_C)[\rho\gamma + r(\rho, d)][d_F + (1 - r)d_w y^d] - \\ (1 - v)[s_F + (1 - s_F)s_C]y^d\end{matrix}\right\} (iv) \end{cases}$$

At the steady state, $\dot{d}_w = \dot{y}^d = 0$

$y^d_{,\dot{d}_w=0}$ phase line expression from equation (*ii*) is :

$g\left((1 - v)y^d, \rho, s_F, d_F\right) = [v(G^{\alpha} + c_0 - 1) - r(G^{\alpha} + c_0 - 1)d_w]/d_w$

$y^d_{,\dot{y}^d=0}$ phase line expression from equation (*iv*) is :

$$\emptyset_1\left(g\Big((1-v)y^d,\rho,s_F,d_F\Big)\right)+(1-s_C)[\rho\gamma+r(\rho,d)][(1-r)d_w y^d]-(1-v)[s_F+(1-s_F)s_C]y^d=0$$

- ***System (IV): equations (i) and (iii)***

$$\begin{cases}\dot{G}=2\mu\left[r\dot{d}_w-\dot{v}\right] & \textbf{(i)}\\ \dot{d}_F=\emptyset_1\left(g\Big((1-v)y^d,\rho,s_F,d_F\Big)\right)-s_F[(1-v)y^d-r(\rho,d)d_F]-g((1-v)y^d,\rho,s_F,d_F)d_F & \textbf{(iii)}\end{cases}$$

At the steady state, $\dot{d}_F=\dot{G}=0$

$G_{,\dot{d}_F=0}$ phase line expression from equation *(iii)* is :

$$\emptyset_1\left(g\Big((1-v)y^d,\rho,s_F,d_F\Big)\right)-g\Big((1-v)y^d,\rho,s_F,d_F\Big)d_F+s_F r(\rho,d)d_F=s_F[(1-v)y^d]$$

$s_F[(1-v)y^d]>0$

$G_{,\dot{G}=0}$ phase line expression from equation *(i)* is:

$$G=\left[\frac{\dot{v}}{r}+(1-c_0)(v-rd_w)+d_w\hat{y}^d+d_w g\Big((1-v)y^d,\rho,s_F,d_F\Big)\right]^{1/\alpha}$$

References

Asada T. and **Franke R., 1994** "A Keynes-Goodwin model of business cycle", Journal of Economic Behavior and Organization 24 (1994), 273-295.

Asada T., 2001 « Nonlinear dynamics of debt and capital: a post-keynesian analysis » Japan Association for Evolutionary Economics/Y. Aruka, Evolutionary Controversies in Economics: A new transdisciplinary approach, Springer-Verlag Tokyo;

Asada T., 2006 "Stabilization policy in a Keynes-Goodwin model with debt accumulation" structural change and economic dynamics 17 (2006) PP 466-485

Atkeson and **Kehoe, 2004**; « Deflation and Depression: is there an empirical link?" federal reserve bank of Minneapolis, Research Department, Staff report 331

Atkinson A. B., Piketty T., and **Saez E., 2010** « Top Incomes in the Long Run History », Journal of Economic Littérature, 2010

Barba A. and **Pivetti M., 2009** "Rising household debt: Its causes and macroeconomic implications – a long period analysis" Cambridge Journal of Economics 2009, 33, 113-137

Barnes, 2007 « Minsky's FIH, information asymmetry and accounting information : the UK financial crises of 1866 and 1987 » SAGE Publications Vol 12 (1) : 29-53

Bernanke, 1983 « Nonmonetary Effects of the Financial Crisis in Propagation of the Great Depression", American Economic Review, Vol. 73(3), June 1983, p. 257-76

Blinder A., 1975 " Distribution Effects and the Aggregate Consumption function" The Journal of Political Economy, Vol. 83, N° 3 (June 1975) pp 447-475; published by The University of Chicago Press.

Charpe M., 2009 "Dette des ménages et instabilité financière", Revue de la régulation Numéro 5.

Eggertsson G. B. and **Krugman P., 2010** "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach" 16th Nov 2010.

Davidson, 2008 " How to solve the U.S. housing problem and avoid a recession: a revised HOLC and RTC" MPRA Paper 7427, University Library of Munich, Germany

Delli Gatti and **Gallegati**, **1995** « Financial fragility and economic fluctuations : keynesian views (and an addendum) Econ Notes 24 :513-554

Duesenberry J. S., 1949 "Income, Saving and the Theory of Consumer Behavior" Cambridge MA, Harvard University Press.

Fisher I., 1933 « The debt deflation theory of great depressions, Econometrica 1 :337-357

Fitoussi J-P. and **Saraceno F. 2010** " Inequality and Macroeconomic Performance", Document de Travail de l'OFCE, OFCE/POLHIA, N° 2010-13,

Franke R. and **Semmler W., 1989** « Debt financing of firms, instability, and cycles in a dynamical macroeconomic growth model » in : Semmler W (ed) Financial dynamics and business cycles : new perspectives, Sharpe, New York, pp 38-64

Goodwin, 1967 "A Growth cycle" In Feinstein CH (ed) Socialism, Capitalism and Economic Growth. Cambridge University Press, Cambridge, pp 54-58

Iacoviello M., 2007 « Household Debt and Income inequality, 1963-2003 » Oct. 2007, Boston college

Kalecki M., (1937) "The principle of increasing risk", Economica 4:440-447.

Keen S., 1995 « Financial and economic breakdown: modeling Minsky's financial instability hypothesis », Journal of Post Keynesian Economics, Summer 1995, Vol. 17, N°4

Keen S., 2000 « The nonlinear economics of debt-deflation » in : Barnett WA, Chiarella C, Keen S, Marks R, Schnabl H (eds) Commerce, complexity, and evolution. Cambridge University Press, Cambridge, pp 83-110

Keynes J. M., 1936 « The General Theory of Employment, Interest and Money", New-York: Harcourt Brace.

Kruger and Perri, 2006 « Does income Inequality lead to consumption inequality? Evidence and Theory », The Review of Economic Studies, vol. 73, 163-93

Minsky H.P., 1975 « John Maynard Keynes » Columbia University Press, new york

Minsky H.P., 1977 "The Financial Instability Hypothesis: an interpretation to Keynes and an Alternative to Standard Theory", Nebraska Journal of Economics, Winter, 16.

Minsky H.P., 1986 « Stabilizing an unstable economy » Yale University Press, New Haven

Moss D. A., 2010 « Comments on bank failure/Regulation/Inequality chart », Harvard Business school, August 2010.

Krugman P., 2010, « Inequality and crises: coincidence and causation? » Conference by Luxembourg Income Study and Fondation Alphonse Weicker on the theme "Inequality and the Status o the Middle Class: lessons from the Luxembourg Income Study" 28th June 2010.

Reinhart C. and Rogoff K., 2010 "Cette fois c'est different. Huit siècles de folie financière", Pearson Education, Paris, France.

Semmler W., 1987 "A Macroeconomic limit cycle with financial perturbations" Journal of Economic Behavior and Organization, September, 8.

Stiglitz J. E., 1969 "Distribution of Income and Wealth among Individuals" Econometrica, Econometric Society, vol. 37 (3), July, pp 382-97.

Stiglitz J. E., 2010 « Le triomphe de la cupidité », Les Liens qui Libèrent, Février 2010.

Taylor L. and **O'Connell S., 1985** "A Minsky Crisis", Quaterly Journal of Economics, Supplement, 100.

Townsend I., 2009 "Economic Policy and Statistics", Library-House of Commons, SN/EP/3870, UK.

Treasury Committee-House of Commons, 2009 « Banking crises : dealing with the failure of UK banks », Seventh report of Session 2008-2009, UK

Von Peter, 2005 « Debt-deflation: concepts and a stylised model », Bank of International Settlement Working Papers N°176

Weber L., 2005 « A contribution to Goodwin's Cycle Model from a system dynamics perspective »), 2005.

Wolfson M. H., 2002 « Minsky's theory of financial crises in a global context", Journal of *Economic* Issues, 36, 2, p. 393-400

Wray L. R., 2008 "Financial Markets Meltdown: What can we learn from Minsky" Economic Public Policy Brief Archive ppb_94, The Levy Economics Institute.