Monetary Policy and the NAIRU in a New-Keynesian New-Growth Model

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Abstract

In New Keynesian models, temporary nominal shocks, like cost push shocks (temporary upward shift of the Philips Curve), generally do not have very persistent effects on employment. This paper introduces endogenous growth along the lines of Romer's famous learning by doing mechanism into a New Keynesian general equilibrium model in order to contribute to the explanation of medium run increases in unemployment and the NAIRU like those observed in continental Europe since the middle of the 1970s. It examines the effects of a cost push shock lasting for two years. Substantial effects (between 1 and 4 percentage points) on employment persist for 50 to 300 quarters while inflation stops falling very soon: The NAIRU increases Furthermore, there is a trade-off between faster disinflation and long-term effects on unemployment.

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1 Introduction

"Short-run macroeconomics and long-run growth theory have never been properly integrated. It is only a slight caricature to say that once upon a time the long run was treated casually as forward extension of the short run, whereas nowadays the tendency is to treat the short run casually as a backward extension of the long-run."

Robert M. Solow/Athanasios Orphanides (1990).[1]

One of the most widespread beliefs in mainstream macroeconomic theory is the separation of short- and long run analysis. While aggregate demand may cause temporary output fluctuations, output ultimately returns to potential as prices adjust. What is more, potential output itself and the "natural-" or "non-accelerating-inflation-rate-of-unemployment" (NAIRU) is not affected. Long-run unemployment is said to depend only on the regulatory setting of the labour and product markets, like trade union power or unemployment benefits, for these factors determine the bargaining power of unions, real wages and thus unemployment. Hence the long run Phillips curve is vertical and monetary- and fiscal policies have no role to play in the long run.

For the last two decades, labour market economists have tried to apply the theoretical relationships outlined above to the steady rise in continental European unemployment since the 1970s. They tried to quantify the effect of labour market institutions and regulations (for instance, employment protection, the tax bite on wages, union members as a percentage of employees, generosity of unemployment benefits, duration of payment of unemployment benefits). The results have not been entirely conclusive. At the outset, labour economist where encouraged by the fact that labour market rigidities seem to be able to explain why unemployment is so much lower in the United States, with its flexible labour market, weak unions etc., than in Europe. However, as it comes to the evolution of European unemployment over time, Blanchard notes that "explanations based solely on institutions also run into a major empirical problem: Many of these institutions where already present when unemployment was low, and, while many became more employment-unfriendly in the 1970s, the movement since then has been largely in the opposite direction."[2] A closer look at the results from multicountry regressions which are often seen as confirming the view that institutions are to blame reveals further problems with this approach, as will be discussed below.

At the same time, mainstream macroeconomics has by and large neglected if not completely ignored the possibility that the persistent increase in unemployment could be the result of changes of changes in productive capacity in response monetary policy, or more generally the handling of shocks hitting the economy by monetary

[1]Solow/ Orphanides (1990), p. 258.

[2]Blanchard/ Wolfers (2000), p. C2.

policy. This is remarkable because simple intuition would suggest that changes in the capital stock are affected by monetary policy via investment, which obviously reacts to changes in real interest rates and macroeconomic conditions more generally. This in turn would be expected to affect labour productivity and thus the real wage employers are willing to pay. Using the terminology of a simple macroeconomics textbook, a restrictive monetary policy could potentially cause to a downward shift of the price setting function relative to the wage setting function. Furthermore, the increase in European unemployment has been accompanied by a slowdown in productivity growth, which perhaps suggests a causal relationship between productivity growth and unemployment.

A notable exception is Blanchard (2000, 1998), who argued that the implementation of high real interest rates by European central banks in the 1980s in order to reduce inflation meant that the marginal product of capital had to increase. With a neoclassical production function, this requires a fall of the capital labour ratio and thus a decline of the marginal product of labour and the real wage. To enforce the later in the face of downward real rigidity, unemployment has to rise.[3] However, while long-term real interest rates have indeed risen in the 1980s as opposed to the 1970s, they have declined in the second half of the 1990s and are now broadly at about the level they were at the end of the 60s. Meanwhile, unemployment in the big European economies remains stubbornly high.

In this paper, we attempt to contribute to the explanation of long swings in the NAIRU by integrating a learning by doing production technology and external technological progress into a New Keynesian modelling framework which features unemployment. These concepts are usually associated with New Growth Theory. We adopt them in a very simple, stylised fashion which uses the capital stock as an indicator of economic activity ("doing") and thus links total factor productivity to the capital stock. The resulting model economy generates endogenous growth due to constant returns to investment and thus presents a response to the above complaint of Solow and Orphanides. We are interested in whether this model is able to produce persistent effects of a disinflation like the one implemented by European central banks during at the end of the 1970s, and whether the hawkishness of the central bank matters, which is captured by the coefficients on inflation and the output gap in its interest rate rule. We therefore subject the economy to an inflationary shock, a so called cost push shock, in order to generate the need for a disinflation which due to the nature of nominal rigidity in the model requires a recession. It turns out the effects on employment can indeed be quite persistent and that unemployment might remain below its steady state value for more than 300 quarters. The fall in demand and the increase in real interest rates depresses investment, which in turn reduces the productivity growth rate and thus the real wage growth rate consistent with stable inflation. With real wages being rigid, this requires higher unemployment: The NAIRU has increased.

[3] See Blanchard (1998), pp. 5-18 and Blanchard (2000), pp. 2-15, and also Bean (1997), p. 95.

The paper is structured as follows: Sections 2 to 4 are some brief discussions of some of the problems which the institutional approach towards explaining the European unemployment problem has run into (Section 2), some empirical evidence on the relationship between the productivity slowdown and the increase in unemployment (section 3) and the relationship between monetary policy and the NAIRU. Section 5 develops the model, while section 6 discusses the calibration. We are using German data to calibrate wage setting and use a recent estimate for the Bundesbank to calibrate the policy rule in the baseline simulation. Section 7 presents and interprets the simulation results. The economy is subjected to cost push shock lasting eight quarters. We vary the policy rule by changing the coefficient on the output gap, and it turns out that a stronger preference on output stabilisation limits the increase in unemployment in the short and in the long run but leads to a stronger acceleration of inflation in the short run. To summarise the trade-off which policymakers face, we compute average unemployment rates, NAIRUs and inflation rates over 110 quarters and the resulting Phillips Curves are downwards sloping. Furthermore, we check the robustness of our results against changes in real wage rigidity and the slope of capital stock adjustment costs. Section 8 offers some conclusions.

2 Certain Pitfalls of the Institutional Approach to explaining European Unemployment

There have been various attempts to verify the institutional hypotheses by regressing unemployment on (indicators of) the institutional variables listed above, and it turns out that the approach runs into problems when trying to explain the dynamics of unemployment. A recent IMF (2003) study using data from 20 OECD economies over a period from 1965 to 1998 concluded that institutions "hardly account for the growing trend observed in most European countries and the dramatic fall in U.S. unemployment in the 90's": The part of the unemployment rate not explained by institutions increases over time.[4] Similarly, Blanchard/Wolfers (2000) noted that "while labour market institutions can possibly explain cross country differences today, they do not appear to able to explain the general evolution of unemployment over time."[5] Looking at particular country, it turns out that institutions are especially weak in explaining the evolution of unemployment in Germany and France, which are perhaps most often cited as examples of "sclerotic" economies.[6]

A study by Nickell (2002, 2005) et al stretching over a slightly shorter time period reveals similar problems in that institutions explain virtually nothing for Western Germany (where unemployment rose from about 1% to about 6%.), Finland and New Zealand. In Spain, unemployment rose from about 2% in 1960 to about 22%

[4] IMF (2003), p. 134.

[5] See Blanchard/ Wolfers (2000), p. C2.

[6] See IMF (2003), pp. 138-141.

in 1995. However, only 5% of this can be explained by institutions. Though this is the most extreme example, the limits of the explanatory power of the institutional variables are obvious for other countries, including Ireland, France, the UK and Italy.[7] Furthermore, for a number of countries, Nickell's model would essentially predict negative unemployment rates in case of unchanging institutions: This sheds doubt on the validity of the model: Maybe the "true" coefficients are sensitive to variations of the independent variables, implying that counterfactual simulations will yield only crude estimates of what would have actually happened had the exogenous variables evolved as assumed for the purpose of the simulation. It also has to be noted that the limited explanatory power of theses models partly rest on the inclusion of lagged unemployment in the regression, with coefficients of 0.79 (IMF) and 0.87 (Nickel et al) respectively. This means that to a large degree, unemployment is explaining itself, or as Nickel et al put it: "This reflects a high level of persistence and/or the inability of the included variables to explain what is going on."[8] A study of 19 OECD countries by Elmeskov et al. (1998) in a paper forming part of the OECD research following up the "Jobs Study" adds to this impression. They use a different approach to assess the quantitative impact of institutions on unemployment by asking how much of the change in structural rather than actual unemployment is accounted for by institutional changes. This makes sense in principle because the conventional view is that structural unemployment (or the NAIRU) is driven by institutions alone. They then compare the change in this structural unemployment rate from 1983 to 1995 to the contributions of the individual institutions and a country specific effect.[9] It turns out that this country specific effect explains most of the change in structural unemployment in almost every country, with the exception of the Netherlands, the UK, Belgium and Ireland, in the later of which it still explains about 50% of the change.[10]

Furthermore, the results from estimations of specifications of the above kind are not very robust across different studies or to adding observations or variables. Baker et. al survey six recent papers[11] and find that labour taxes and benefit duration are significant in all studies were they are included, and the replacement rate in all but one. However, the effect a 10 percentage point increase taxes and the replacement rate on unemployment ranges from 0.9 p.p. to 2.1 and from 0.1-1.3 p.p., respectively. The effect of an increase in benefit duration by one year ranges from 0.7% to 1.4%.[12] Baker et.al. also report that an earlier version of the Nickel et al paper covering a period shorter by three years produced very different estimates of the coefficients of labour

[7] See Nickell (2002), pp. 44-45.

[8] See Nickel et al (2005), p. 15.

[9] See Elmeskov et. al (1998). The country specific effect is the difference between the structural unemployment rate and the institutional variables times their respective coefficients.

[10] See Elmeskov et. al (1998), p. 220, Table 3a.

[11] (Nickel 1997, Elmeskov et al 1998, Belot/van ours 2002, Nickel et al 2002, Blanchard/Wolfers 2000, Bertola et. al. 2001)

[12] See Baker et al (2002), pp.43-44.

taxes, benefit duration, and bargaining coordination. In the paper's final version, their (long-run) impacts are reduced by more than 30%, 50% and 40%, respectively.[13] Nickel et al's results are apparently very sensitive to the inclusion of additional data. Finally, Belot/ van Ours (2004) find that the significance of institutional variables is extremely sensitive to the inclusion of time and country fixed effects.[14]

One of the crucial the underlying assumptions of panel data regressions of unemployment on labour market institution is that labour market institutions are exogenous and are not affected by those force which are affecting unemployment or by unemployment itself. This assumption might be violated with respect to the tax wedge, as rising unemployment increases expenditures on transfers and erodes the tax base. The generosity and duration of unemployment benefits might as well be increased as a response to rising unemployment.

The attitude of many researchers in the field to this problem seems to be a somewhat relaxed one, as it is rarely mentioned as a problem.[15] The Elmeskov et a. (1998) study discussed above does include a test of whether benefit generosity and the tax wedge do Granger cause unemployment. A variable x is said to Granger cause a variable y if in a regression of y on lagged values of x, the number of lags being determined by suitable criteria, the lagged values of x are jointly significant.[16] They conduct the test separately for each country, with the data ranging from 1970 to 1995. They find that unemployment Granger causes benefit generosity in Belgium, France, Italy, the UK, the United States and the Netherlands, though in the later the result is significant only at the 10% level. This is an interesting result because the list includes both high and low unemployment countries where benefit levels have been moving in opposite directions. The tax wedge is found to Granger cause unemployment in Austria, Ireland and Norway.[17] Hence there is some evidence that causality could run both ways.

3 Productivity Growth and Unemployment

As mentioned above, this paper aims to explain long swings in unemployment without recurring to changes in labour market institutions. Instead, the model developed below aims to analyse how a contractionary monetary policy can reduce productivity growth by reducing investment, thus increasing the NAIRU if real wage growth is

[13] See Baker et. al (2003), p. 35. These numbers can be easily checked by comparing the coefficients for the 2001 version of the Nickel et al study reported in Baker et al, p.47, fourth column of the table, with the coefficients reported in Nickel (2002), p. 37, column 2 of the table, or in Nickel (2005), column 3, after modifying them for the effects of the long-run multiplier. Sadly, it is not possible to get hold of the earlier version of the Nickel paper.

[14] See Belot/Van Ours (2004), p. 635.

[15] See for instance Nickell et al (2002), p.2, or IMF (2003).

[16] See Gujarati (2004), p. 698.

[17] See Elmeskov et. al. (1998), pp. 248-249.

rigid. While real wage rigidity might well be influenced by labour market institutions, and thus institutions would affect the evolution of unemployment, the actual cause would lie in the behaviour of monetary policy, or in the response of monetary policy to a shock. This section discusses some evidence on the relationship between productivity growth and the NAIRU, and between the NAIRU and monetary policy.

To get an impression of how changes in productivity growth rates are associated with NAIRU movements, Figures 7 to 12 plot the NAWRU and the trend growth rate of labour productivity per hour worked, as estimated by Skoczylas/ Tissot (2005) for Western Germany, Spain, France, Italy, the United States and the United Kingdom. They employ two different ways to account for cyclicality, which is why there are two lines measuring trend productivity in each graph.[18]

The graphs show that there is not an always perfectly tight, but still apparent relationship between productivity growth the NAIRU. Looking at the figures for Germany, France, Spain and Italy, it is obvious that the increases in the NAIRU have taken place during times when productivity growth slowed down. For instance, in Germany (Figure 7) the acceleration in NAWRU growth at the beginning of the 1970s very much coincides with the productivity slowdown. After having crept up by about 0.36 percentage points from 1965 to 1971, during the seventies, the NAIRU increases at a rate of about 0.38 percentage points per year, while at the same time productivity growth shifts at least twice by an accumulated 3.2 to 4 p.p., respectively. In about 1983, four years after the last negative break, the NAWRU decelerates and reaches a plateau, and appears to bend downwards. Indeed the HP filtered series shows a small positive break in 1988, while the baseline estimate shows a positive break in 1991 (not shown). In the case of France (Figure 9), a similar picture emerges, with a rising NAWRU being accompanied by a continuous slowdown in productivity growth. At the beginning of the nineties, the slowdown in productivity growth stops, and the NAWRU seems to reach a plateau at about the same time. At the end of the sample, there is some evidence about an acceleration of productivity growth and a turnaround in the NAIRU at around 2000. In fact, Frances NAWRU has continued to decline to a value of about 9% in 2005(not shown). Turning towards Spain, Italy and the UK (Figures 8, 10 and 12), NAWRU increases are clearly associated with massive reductions in productivity growth here as well. However, the relationship is less tight in the other direction: The turnarounds in those countries, especially the massive turnaround observed in Spain, is not associated with an acceleration in productivity. The graph for the United States (Figure 11), whose NAIRU moves by much less to begin with, does show some association of both increases and decreases in the NAIRU with changes in productivity growth.

There has been some tighter evidence that changes in productivity growth affect unemployment as well. An early example of this has been Bruno/ Sachs (1982), who argue that a labour productivity slowdown which was anticipated by workers wage

[18] see Skocylas/ Tissot (2005), pp. 11-12 for discussion of the advantages and disadvantages of HP-Filtering.

demands caused unemployment in British manufacturing to increase.[19] Productivity growth or total factor productivity growth are sometimes controlled for in regressions aiming to assess the impact of labour market institutions. For instance, in the IMF study cited above, a one p.p. reduction in productivity growth increases unemployment by 0.32 percentage points, while the Nickel study, while the Nickel et al study cited above finds that a 1 percentage point decrease in total factor productivity causes a 1.28 p.p. increase in unemployment.

There have also been studies explicitly aimed at explaining the evolution of unemployment by macroeconomic shocks like shifts in productivity growth or the real interest rate, while trying to explain cross country differences by differences in labour market institutions. Fitoussi et al (2000) examine 19 countries using annual data stretching from 1960 to 1998 asking whether productivity growth (where productivity here means output per worker), the "world" real interest rate (which is an average of the G7 countries long run real interest rate) and oil prices can explain unemployment. The effect of a one percentage point reduction of productivity growth are allowed to differ from country to country in order to capture the possibility that countries with a more rigid labour market would suffer more from shocks. For Germany, the equation predicts a 0.79 percentage point increase in unemployment, while for France, Italy or Spain the effect would be as high as 1.6, 1.22 or 2.22 p.p.[20] This is quite substantial given the massive slowdown in labour productivity in these countries. The role of institutions is however less clear, because among the countries with higher country specific coefficients are for instance the United States, the United Kingdom, the Netherlands or New Zealand who are conventionally thought to have quite flexible labour markets.

Blanchard/ Wolfers (2000) estimate a specification which explicitly models the interactions of shocks and institutions, i.e. institutional variable effectively become part of the coefficient on the shocks. The shocks include TFP growth, the long run real interest rate and a measure of labour demand, while the institutions considered are the replacement rate as measured in Nickell (1997), benefit duration (in years), employment protection (simple ranking from 1 to 20), the tax wedge as in Nickel et al (2002) and measures of union contract coverage, union density and bargaining coordination.[21] Both shocks and institutions are significant, though concerning the later this finding is not robust against variations in the way the variables are measured.[22] A one p.p. reduction in TFP growth increases unemployment by 0.71 p.p. if institutions are at the sample average. More employment unfriendly institutions cause the shock to have higher effects, so that the model can explain both cross country differences and the evolution of unemployment over time.

[19] See Bruno/ Sachs (1982), p. 700/701.
[20] See Fitoussi et al (2000), pp. 247 to 250.
[21] See Blanchard/Wolfers (2001), p. C19.
[22] See Blanchard/Wolfers (2000), p. C31.

4 Evidence on the Effect of Monetary Policy on the NAIRU

In two papers, Ball argues the change in the NAIRU during the 1980s can be explained by the monetary policy stance. His theoretical motivation is theories of hysteresis in the labour market like the model proposed by Blanchard/ Summers (1986). In those models, worker who become unemployed have a lower effect on wage bargaining than those who are still employed (the "insiders"). Furthermore, an extended unemployment spell might reduce job search activity. These effects reduces the downward pressure on real wages which the unemployed would otherwise, and increase the NAIRU. The unemployment benefit system plays an obvious role for the degree of hysteresis.

Ball measures the stance of policy during that period indirectly by the behaviour of inflation (Ball (1996)), and directly by examining the evolution of real interest rates (Ball(1999)). He focuses on 20 OECD countries and follows the OECD Jobs study in constructing his NAIRU data.[23] In his first paper, Ball uses two measures of inflation dynamics: The size of the disinflation from 1980 to 1990 and the length of the longest disinflation during that period. Those matter because the former is related to the size of the unemployment increase, while the latter indicates for how long the actual unemployment rate exceeded the NAIRU.

Ball finds that while the length and the size of disinflation explain a substantial share of the increase in the NAIRU over the ten year period, large prediction errors remain. Therefore he examines the interaction between benefit duration and the policy stance does a better job at explaining the rise in the NAIRU than considering both on their own for the reasons given above.[24] Hence Ball considers interactions of benefit duration with the inflation decrease and squared disinflation length. Simple regressions for both variables yield fits substantially superior to when the policy variables are not interacted with benefit duration, especially for the change in inflation. The specifications Ball runs differ in which of the two inflation variables are interacted with benefit duration, but all yield an R^2 between 0.67 and 0.75. [25] Ball then subjects this procedure to a series of robustness experiments, all of which basically confirm the original conclusions, namely that the policy stance explains major parts of the increase in the NAIRU and that fit substantially improves if the policy stance is interacted with benefit duration. [26]

[23] The approach is fairly straightforward. Assume relationship between the change in inflation and unemployment U of the following form: $\pi_t - \pi_{t-1} = a(U_t - U_t^*)$, where U^* denotes the natural rate of unemployment. This is an equation in two unknowns: U^* and a. The equation is then iterated forward, which produces another equation with the same unknowns, so that U^* and a can be computed. The process is then repeated with observations from t+1 and t+2, and so on and so forth. See Ball (1996), p.3.

[24] See Ball (1996), p. 13.

[25] Ball (1996), p. 12.

[26] See Ball (1996), pp. 13-15.

Ball then turns towards the role of monetary as measured by interest rates during the recessions of the early 1980s, and tries to explain the subsequent evolution of the NAIRU by the extent of policy easing in response to those recessions. He uses annual data and defines a recession as one or more consecutive years of growth below one percent a year, while the policy stance measured by largest cumulative decrease of short term real interest rates in any part of the recession's first year, or the average of the largest cumulative decreases from each recession in case the country had two recessions.[27] Ball then uses this measure of policy and benefit duration to explain two variables: the change in the NAIRU from the peak before the first recession until five years after the peak, and this change divided by the change in actual unemployment over the same time period. The later variable is called degree of hysteresis and accounts for the fact that the severity of recessions and thus the increase in actual unemployment vary over the sample and hence one would observe different increases in the NAIRU even if actual unemployment fed into the NAIRU to the same extent in all countries, i.e. if monetary policy and benefit duration had been the same.[28] Fit is indeed substantially better when the degree of hysteresis is used as a dependent variable, with an adjusted R^2 of 0.62 as opposed to 0.43. Concerning the quantitative impact of the two variables on the degree of hysteresis, "The coefficient on maximum easing implies that raising that variable from 0 to 6 (Sweden's value, the highest in the sample) reduces the degree of hysteresis by 0.54. Reducing the duration of unemployment benefits from indefinite to half a year reduces the degree of hysteresis by 0.35. Thus policymakers can reduce hysteresis through both macroeconomic and labour market policy, and the former has somewhat larger effects."[29]

Ball also tries to explain reductions in the NAIRU in OECD countries by referring to the stance of monetary policy relative to the situation of the macro economy, and finds that to some extent, monetary policy can also explain NAIRU reductions.

5 The Model

While the previous section argues that there exists empirical evidence which links the monetary policy stance to the subsequent evolution of the NAIRU, this section develops a dynamic general equilibrium model which can explain why that might be the case. It aims to explain broad movements of unemployment, the NAIRU and productivity growth by the response of monetary policy to a cost-push shock, which might for instance be thought of as an Oil price shock. Just like in Balls empirical analysis of the 80s, the scenario is that the central bank wants to disinflate the economy and thus raises interest rates to contract demand.

[27] Ball notes that his dating criterion for recessions yields only two countries with two recessions and thus is stricter than the one used with quarterly data. See Ball (1999), p. 205.

[28] See Ball (1999), p. 205-206.

[29] Ball (1999), p. 207.

The next subsection explains the learning by doing production set up and the motivation behind it. Then the model is developed: section 5.2 deals with the household sector and how the households preferences over effort yield a function relating effort to various variables, including the workers wage. Section 5.3 shows how firms make use of the effort function when minimising their production cost by paying efficiency wages. This introduces unemployment into the model. Furthermore, it shows how in an endogenous growth economy marginal costs are affected by capital accumulation in a much stronger way than with a neoclassical production technology. Section 5.4 shows how capital goods production is organised, while section 5.5 introduces sticky prices via quadratic price adjustment costs. Section 5.6 summarizes aggregate equations to highlight some of the mechanisms which will become important in the simulations and for the convenience of the reader when dealing with the simulations in section 7.

5.1 The knowledge Spill over setup

The basic idea in the knowledge spill over model is to start off with a standard neoclassical production function with labour augmenting technical progress,

$$Y_t(i) = F(K_t(i), A_t(i)n_t(i)) \tag{1}$$

Romer then makes two crucial assumptions:

- There is learning by doing in the economy: Increasing production causes firms to learn how to produce more efficiently. This idea was first suggested by Arrow(1962). In the Romer setup, net investment is used as a proxy for activity, in a fashion such that $A_t(i)$ is proportional to the firm's capital stock.
- Knowledge is a public good. Hence each firm's knowledge is in fact proportional to the aggregate capital stock rather than to its own.[30] However, the impact of the firms capital stock on the aggregate capital stock is so small that they can be neglected.

Thus the production function becomes

$$Y_t(i) = F(K_t(i), K_t n_t(i)) \tag{2}$$

This implies that there are now constant returns to capital at the economy wide level, allowing per capita output to grow. However, there are still decreasing returns to capital at the firm level. In the Romer model, where the labour force is in elastically supplied and wages are perfectly flexible, this leads to an inefficiently low choice of the capital stock. In turn, this leads to a growth rate which is inefficiently low because

[30] See Barro/ Sala-i-Martin (2004), pp.21-22.

saving is to low as the individual return on capital (which decreases in the capital stock) falls short of the social return on capital (which is independent of the level of the capital stock).

We are completely aware that this model captures the notion of learning by doing in a very simplistic fashion and remind the reader that this paper is a first pass at introducing endogenous growth into a monetary macroeconomic model. Note however that the steady state in the learning by doing model satisfies the famous five stylised facts of growth: Output per capita and capital per labour keep increasing, the capital output ratio is trend less, the real wage per unit of labour keeps increasing, the rate of profit is trend less and the share of GDP going to capital and labour are trend less as well. This will also be true for the model developed below. Thus from an empirical point of view, there is no reason rendering the neoclassical production function superior to the alternative employed here.

Furthermore, as will be further stressed below, an expansion in aggregate employment will tend to increase the productivity growth rate, because it increases the marginal product of capital and thus investment. This is an effect which will become important if the above technology is introduced into a business cycle model featuring fluctuations in employment.

5.2 Households

Danthine Kurman/ Kurmann (2004) show how to introduce unemployment in a general equilibrium model without moving away from the representative agent framework. In the Danthine-Kurmann setup (later on referred to as "DK"), individuals are organized in families in a zero-one continuum of families which are infinitely lived. All decisions regarding the intertemporal allocation of consumption are made at the family level. Each family supplies one unit of labour in elastically. In each period, the share of unemployed members is the same for each family. In addition, in this paper, some workers will supply overhead labour, whose nature will be described in more detail below. Overhead workers never get unemployed because no firm can produce without a certain amount of overhead staff. It is assumed that each family has the same amount of overhead workers.

These assumptions allows to have unemployment in the model while at the same time maintaining homogeneity of the agents who make the consumption decision (which are families, not individuals). The individuals, however, derive disutility from the effort they supply in their job, which determines the efficiency of a labour unit in the production process. More formally, a household in the DK model maximizes

$$U = E_0 \sum_{i=0}^{\infty} \beta^i \left[u(C_{t+i}) - (n_{t+i} - \overline{n}) \, G(e_{t+i}) \right], u\prime > 0, u\prime\prime < 0. \tag{3}$$

$$s.t. \; n_{t+i} w_{t+i} + \frac{B_{t+i-1}}{P_t}(1 + i_{t-1+i}) + \Gamma_{t+i} \geq C_{t+i} + \frac{B_{t+i}}{P_{t+i}} \tag{4}$$

where $n_{t+i}, \overline{n}, w_{t+i}, B_{t+i-1}, i_{t-1+i}, \digamma_{t+i}$ and C_{t+i} refer to total employment (the total labour force is normalised to one), overhead employment, the real wage, bond holdings at the end of period t-1, the interest rate on savings from period t-1, firms profits, a CES consumption basket, i.e. $C_t = \left[\int_0^1 (c_t(i))^{\frac{(1-\theta)}{\theta}} di\right]^{\frac{\theta}{1-\theta}}$, and the effort function of individual j is of the form

$$\begin{aligned} G(e_t(j) &= (e_t(j) - (\phi_0 + \phi_1 \log w_t(j) + \phi_2(n_t - \overline{n}) + \phi_3 \log w_t + \phi_4 \log w_{t-1}))^2, \quad (5) \\ \phi_1 &> 0, \phi_2, \phi_3, \phi_4 < 0, \phi_1 > -\phi_3 \end{aligned}$$

Note that the effort function enters the families utility separately which implies that it is independent of the budget constraint. The first order conditions with respect to consumption and effort imply the following two equations

$$u'(C_t) = \beta E_t \left[u'(C_{t+1}) \frac{1}{1+\pi_{t+1}}\right] [1 + i_t] \quad (6)$$

$$e_t(j) = \phi_0 + \phi_1 \log w_t(j) + \phi_2 (n_t - \overline{n}) + \phi_3 \log w_t + \phi_4 \log w_{t-1}. \quad (7)$$

The first of those is the familiar consumption Euler equation, while the second determines the optimal effort level. The structure of the effort function is motivated by the idea of "gift exchange" between the firm and the worker. The worker feels he has to be compensated for a higher effort level with a higher wage. Accordingly, a higher contemporary average wage w_t reduces effort because it represents a "reference level" which the current employers wage offer is compared with, and a higher average past real wage w_{t-1} boosts the workers aspiration as well.[31] Finally, the aggregate employment level of non-overhead workers $(n_t - \overline{n})$ summarizes labour market tightness and is thus positively related to the workers outside options, and thus also tends to reduce effort.

Consumers spread their consumption over the various goods in the basket C_t in a cost minimising fashion, i.e. they solve the intra-temporal minimisation problem

$$\begin{aligned} &\min_{c_t(i)} \int_0^1 p_t(i) c_t(i) di \quad (8) \\ s.t. C_t &= \left[\int_0^1 (c_t(i))^{\frac{(1-\theta)}{\theta}} di\right]^{\frac{\theta}{1-\theta}} \end{aligned}$$

Optimal demand for good i resulting from this optimisation is given by $c_t(i) = C_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta}$, where P_t denotes the price index of the consumption basket.

[31] See Danthine/ Kurmann (2004), pp. 111-113.

5.3 Cost Minimisation and Efficiency Wages

The production technology is of the form $Y_t(i) = AK_t(i)^{\alpha}(K_t e_t(i)\,(n_t(i) - \overline{n}))^{1-\alpha}$, which is essentially a Cobb-Douglas production function augmented to allow for learning by doing and the effect of effort level affects on the efficiency of labour. Furthermore, it features overhead labour $\overline{n}$. This is the part of the firms staff the employment of which will not depend on the amount of output produced. Examples of this would be for instance the accounting or controlling division of the firm, or parts of the management. Accordingly, $n_t(i) - \overline{n}$ is the amount of productive labour.

In the Danthine/ Kurman model (2004), in a first stage the firm minimises its cost of producing a given amount of output. To do so it hires capital in an economy wide market and furthermore decides on the wage it is going to pay, taking into account the relationship between effort and wages given by (7).[32] Hence the firm's problem is:

$$
\begin{aligned}
r_t^k K_t(i) + w_t n_t(i) \; s.t. \; Y_t(i) &= AK_t(i)^{\alpha}(K_t e_t(i)\,(n_t(i) - \overline{n}))^{1-\alpha} && (9)\\
\text{and } e_t(i) &= \phi_0 + \phi_1 \log w_t(i) + \phi_2 \log n_t + \phi_3 \log w_t + \phi_4 \log u_{t-1} && (10)
\end{aligned}
$$

by appropriately choosing $K_t(i)$, $n_t(i)$, $w_t(i)$ and $e_t(i)$ as the firm is conscious of the relationship between effort $e_t(i)$ and wages. This yields for capital and labour the F.O.C.s

$$
\begin{aligned}
r_t^k &= \alpha mc_t \frac{Y_t(i)}{K_t(i)} && (11)\\
w_t(j) &= (1-\alpha) mc_t \frac{Y_t(i)}{n_t(i) - \overline{n}}
\end{aligned}
$$

were mc_t and r_t^k refer to real marginal costs and the capital rental rate, which is the price at which the capital stock, is traded, respectively. The capital stock is predetermined each period and is production will be dealt with in the next section. It will be shown below that even though all firms set the wage individually, firms will find it optimal to set the same wage. This then means that the capital -(productive) labour ratio, the output per unit of productive labour ratio, the output per unit of productive labour ratio and marginal costs are the same in all firms, as can be easily verified by dividing the two FOCs and using the properties of the Cobb-Douglas production function. With all firms having the same capital to productive labour ratio, we can furthermore write the production function as $Y_t = A(n_t - \overline{n})^{1-\alpha} K_t$, which makes the fact that at the economy wide level, we have constant returns to capital very obvious. Using these properties we can write

$$
\begin{aligned}
r_t^k &= \alpha mc_t \frac{Y_t}{K_t} = \alpha mc_t A(n_t - \overline{n})^{1-\alpha} && (12)\\
w_t &= (1-\alpha) mc_t \frac{Y_t}{n_t}
\end{aligned}
$$

[32] See Danthine/ Kurman (2004), pp. 114-115.

Note that unlike in a neoclassical economy, at the aggregate level, the marginal product of capital does not depend on the capital labour ratio. This makes sense given that the average product of capital is independent of the stock of capital as well. Note also that because $\alpha < 1$, the marginal product of capital falls short of the average product, because as described above, firms are not internalising the effect of capital accumulation on overall efficiency.

The F.O.C.s with respect to effort and wages are

$$\begin{aligned} n_t(i) &= \frac{\zeta_t \phi_1}{w_t(i)} \\ \zeta_t &= (1-\alpha) mc_t \frac{Y_t(i)}{e_t(i)} \end{aligned} \tag{13}$$

Combining those with the first order condition with respect to labour yields an optimal effort level equal to ϕ_1. Substituting this back into the effort function (7) and noting that, as the firms wage depends only on aggregate variables which are the same for all firms, it must indeed hold that $w_t(i) = w_t$ yields the wage setting relation:

$$\log w_t = \log w_t(i) = \frac{\phi_1 - \phi_0}{\phi_1 + \phi_3} - \frac{\phi_2}{\phi_1 + \phi_3} (n_t - \overline{n}) - \frac{\phi_4}{\phi_1 + \phi_3} \log w_{t-1} \tag{14}$$

Hence with the coefficient restrictions imposed above, the wage depends positively on the past real wage and non-overhead employment. It will be above its market clearing level und thus there is unemployment in the economy.

Equation (14) could be solved for a long run real wage if $-\frac{\phi_4}{\phi_1+\phi_3} < 1$. As mentioned above however, in our model, unlike in the Danthine/ Kurmans, is a growth model, and so the real wage must be growing in the steady state. Therefore a function relating the wage level to employment is not appropriate unless one includes productivity growth as an additional argument. A major driving force of the results of this paper however is that wages are not indexed perfectly indexed to productivity growth. The easiest way to deal with the issue therefore seems to set $-\frac{\phi_4}{\phi_1+\phi_3}$ equal to one, which means that we have real wage growth function, or real wage Phillips curve:

$$\log w_t - \log w_{t-1} = a + b * (n_t - \overline{n}), \text{ with } a = \frac{\phi_0 - \phi_1}{\phi_1 + \phi_3} \text{ and } b = -\frac{\phi_2}{\phi_1 + \phi_3} \tag{15}$$

One can also derive the following expression for real marginal costs as a function of factor prices and the aggregate capital stock (after substituting out the individual firm's capital labour ratio):

$$mc_t = \frac{\left(r_t^k\right)^\alpha w_t^{1-\alpha}}{A\alpha^\alpha (1-\alpha)^{1-\alpha} (\phi_1 K_t)^{1-\alpha}} \tag{16}$$

This shows that with an economy wide capital market, all firms will face the same marginal costs. Furthermore, an expansion of the aggregate capital stock for given values of r_t^k and w_t will reduce each firm's marginal costs. This is because an increase in the capital stock increases overall efficiency, even though firms are not internalising this effect. In the event of a monetary contraction, a contraction or just slower growth of the capital stock during a recession will tend to offset the disinflationary effect of slower real wage growth. By contrast, with a standard neoclassical production function, K_t would not show up in the numerator, and marginal costs would be a function of factor prices alone. In such a conventional setup, an increase in the capital stock would still tend to lower marginal costs by reducing r_t^k, but the effect would be substantially lower.

It remains to determine the size of the overhead labour force. Following Rotemberg/ Woodford (1999), it is assumed that in the steady state, all profit generated by employing productive labour and capital goes to the overhead staff so that the firm ends up with zero profits.[33] This is justified because setting up production is impossible without overhead labour and the firms profit is thus essentially equal to the collective marginal product of its overhead staff. We assume that the overhead staff splits this profit equally. As mentioned above, it is assumed that there is full employment among overhead workers and that the amount of overhead workers required and employed is such that the real wage for overhead and non-overhead workers will be exactly the same in the steady state. These assumptions allow for a straightforward way to determine the amount of overhead and non-overhead workers as a function of total employment: Zero profit requires

$$\frac{\mu - 1}{\mu} Y_t - w_t \overline{n} = 0$$

where $\frac{\mu-1}{\mu}$ is the share of firms profits in output. Substituting $w_t = (1-\alpha)\frac{1}{\mu}\frac{Y_t}{n_t - \overline{n}}$ gives after some manipulation

$$\frac{\mu - 1}{1 - \alpha} = \frac{\overline{n}}{n_t - \overline{n}} \equiv \overline{s}$$

which is the ratio of overhead labour to productive labour, which we call $\overline{s}$. Using $n_t = \overline{n} + (n_t - \overline{n})$, we arrive at

$$\begin{aligned} n_t - \overline{n} &= \frac{n_t}{1 + \overline{s}} \\ \overline{n} &= \frac{\overline{s}}{1 + \overline{s}} n_t \end{aligned} \tag{17}$$

which gives the amount of productive and overhead labour as a function of employment.

[33] See Rotemberg/ Woodford (2004), pp. 15-16.

5.4 Investment

Capital goods are produced by a capital goods producing sector which combines capital and the output good to produce new capital goods. Following Woodford, investment goods production technology relates the next periods capital stock to this period's investment spending in the following way:

$$I_t = \Phi\left(\frac{K_{t+1}}{K_t}\right) K_t \tag{18}$$

where I_t refers to investment expenditures. $\Phi\left(\frac{K_{t+1}}{K_t}\right)$ is a convex function, which reflects capital stock adjustment costs: the marginal cost of increasing the capital stock from period t to period t +1 are increasing. It also has the following properties: $\Phi(1) = \delta$ and $\Phi'(1) = 1$, which reflect that the capital stock is used up at a rate δ and that the first unit of additional capital costs one unit of the output good, i.e. there are no adjustment costs for the first unit of capital goods.[34]

The market is organised as follows: At the beginning of the period, the capital goods producing sector sells all its capital to the output goods producing sector at price Q_t. The output good producing sector produces and then sells the capital stock back to the capital goods producing sector at price $\overline{Q}_t$, so that the difference between the 2 is equal to the rental rate r_t^k which is paid by the goods producing sector i.e. $Q_t - \overline{Q}_t = r_t^k$. The fact that the capital goods producing firm sells all its capital at the beginning of the period is admittedly not very realistic, but it greatly simplifies the first order condition because it prevents the firm from engaging any planning stretching over more than two periods. The maximisation problem of the capital producing firm is accordingly given by

$$\max_{K_t, K_{t+1}} \frac{1}{1+i_t} E_t[\frac{P_{t+1}}{P_t} Q_{t+1}] K_{t+1} - \Phi\left(\frac{K_{t+1}}{K_t}\right) K_t - K_t \overline{Q}_t \tag{19}$$

where i_t and P_t denote the risk free rate and the price level of the output good, respectively. The FOCs with respect to K_{t+1} and K_t are, respectively

$$\begin{aligned} \frac{1}{1+i_t} E_t[\frac{P_{t+1}}{P_t} Q_{t+1}] - \Phi'\left(\frac{K_{t+1}}{K_t}\right) &= 0 \\ \Phi'\left(\frac{K_{t+1}}{K_t}\right) \frac{K_{t+1}}{K_t} - \Phi\left(\frac{K_{t+1}}{K_t}\right) - \overline{Q}_t &= 0 \end{aligned} \tag{20}$$

These two conditions jointly determines $\overline{Q}_t$ and K_{t+}, given i_t, the expectation of $P_{t+1}Q_{t+1}$, and K_t. Note that accordingly Q_t is then simply determined by the identity $Q_t - \overline{Q}_t = r_t^k$. r_t^k is already determined by equation (12).

[34] See Woodford (2003), p. 354.

The following functional form will be assumed for the investment goods production function: $\Phi(\frac{K_{t+1}}{K_t}) = \frac{1}{\gamma}(\frac{K_{t+1}}{K_t})^\gamma - (\frac{1}{\gamma} - \delta)$. As can be easily checked, it has all the desired properties.[35]

Furthermore, it is illuminating to substitute the K_{t+1}FOC into the K_t FOC, which yields

$$\frac{E_t[P_{t+1}Q_{t+1}]K_{t+1}}{1+i_t} - P_t I_t - P_t K_t \overline{Q}_t = 0 \tag{21}$$

which is just saying that capital goods producing firms should make zero expected profits in equilibrium, as it should be with a constant returns to scale capital goods production function and perfect competition in capital goods markets.

5.5 Price Setting and nominal rigidities

Each firm produces one of the variants of the output good in the CES basket. Given that investment expenditure stretches over these variants in precisely the same way as consumption demand, we can write $y_{t+i}(j) = Y_{t+i}\left(\frac{p_{t+i}(j)}{P_{t+i}}\right)^{-\theta}$. It is assumed that the representative firm faces costs if it alters its individual price inflation from a reference level $\Pi - 1$. These costs arise because deviating from the "standard" level of inflation requires the firm to engage in a reoptimisation process which has to be carried out by high paid marketing professionals, while small price changes can be decided by lower paid "frontline" staff. Apart from that, customers dislike price volatility because it requires them to switch between products, which the firm has to compensate by extra marketing efforts, special offers etc. These costs are likely to increase in the firms output as well. Following Lubik/Marzo (2007), we assume the following functional form:

$$AC_{t+i}(j) = \frac{\varphi}{2}(\frac{p_{t+i}(j)}{p_{t+i-1}(j)} - \Pi)^2 y_{t+i}(j)^{36} \tag{22}$$

Demand for the firm's product is as follows: $y_{t+i}(j) = Y_{t+i}\left(\frac{p_{t+i}(j)}{P_{t+i}}\right)^{-\theta}$.The firm j chooses its price $p_{t+i}(j)$ in order to maximise

$$\sum_{i=0}^{\infty}\rho_{t,t+i}\left[\frac{p_{t+i}(j)}{P_{t+i}}y_{t+i}(j) - mc_{t+i}y_{t+i}(j) - AC_{t+i}(j)\right] \tag{23}$$

[35] The way the production of capital goods is introduced into the model follows Bernanke et. al. (1999), p. 1356 to 1357, but slightly modifies their setup by assuming that the capital goods producing sector remains in possession of the capital stock from period t to t+1. This is necessary allow the goods producing sector to rent capital "on spot" as assumed in the previous section.

[36] Our specification differs in that we assume the firms price adjustment costs to depend on its own output rather than on aggregate output, which is clearly more intuitive. Conveniently, it also yields the same result after linearising as Lubik/Marzos specification. See Lubik/Marzo (2007) p. 19.

where $\rho_{t,t+i}$ denotes the discount factor used to discount real profits earned in period t+i back to period t. Note that because households own the firms, we have $\rho_{t,t+i} = \beta^i \frac{u'(C_{t+i})}{u'(C_t)}$. Differentiating with respect to $p_t(j)$ and noting that, as all firms are the same, $p_t(j) = P_t$ holds ex post yield

$$(1-\theta) + \theta mc_t - \varphi\left(\frac{P_t}{P_{t-1}} - \Pi\right)\frac{P_t}{P_{t-1}} + \theta\frac{\varphi}{2}(\frac{P_t}{P_{t-1}} - \Pi)^2$$
$$+E_t\left[\rho_{t,t+1}\varphi\frac{Y_{t+1}}{Y_t}\left[\left(\frac{P_{t+1}}{P_t} - \Pi\right) + \frac{\theta}{2}\left(\frac{P_{t+1}}{P_t} - \Pi\right)^2\frac{P_{t+1}}{P_t}\right]\right] = 0 \tag{24}$$

which is a nonlinear Philips curve. Note that this equation contains no trended variables if the central bank is assumed to target zero inflation (or inflation close to zero): $\frac{Y_{t+1}}{Y_t}$ is output growth plus 1 which will differ from the steady state only by relatively small amount (recall that we are dealing with quarterly data) and similar things will be true for mc_t and the discount rate. Hence linearising (24) will only result in limited distortions of the models results. This yields, making extensive use of the fact that in the steady state $\frac{P_{t+1}}{P_t} = \frac{P_t}{P_{t-1}} = \Pi = 1$

$$\pi_t = \frac{(\theta - 1 - 1)}{\varphi}\widehat{mc}_t + \overline{\rho}(1+g)E_t\pi_{t+1} \tag{25}$$

where $\widehat{mc}_t, \overline{\rho}$, and $\overline{g}$ refer to $\ln mc_t - \ln \mu^{-1}$, where $\mu^{-1} = \frac{\theta-1}{\theta}$ denotes the steady state value of marginal costs, the steady state discount factor and the steady state output growth rate, respectively.[37] This is the standard New Keynesian Philips curve, which relates inflation in period t to the deviation of marginal costs from its steady state value and expected future inflation. It is, however, a consistent feature of empirical estimations of Phillips curves that specifications which include lagged inflation ("hybrid" Phillips curves") perform better than those which include only expected next periods inflation because inflation has inertia.[38] Backward looking elements are easily introduced into the price setting considerations of the firm by assuming that the reference level of inflation is not constant over time but equals last periods inflation, i.e. $\Pi = \frac{P_{t-1}}{P_t}$. If the inflation rate becomes higher for several periods, firms will mandate frontline staff to handle price increases of that size in order to keep costs low, and customers will get used to the different pace of price changes as well. Linearising then yields

$$\pi_t = \frac{\pi_{t-1}}{1+(1+g)\overline{\rho}} + \frac{(\theta-1)\widehat{mc}_t}{\varphi\left(1+(1+g)\overline{\rho}\right)} + \frac{(1+g)\overline{\rho}}{1+(1+g)\overline{\rho}}E_t\pi_{t+1} \tag{26}$$

[37] As usual in models of monopolistic competition where the demand for single product varieties is of the Dixit-Stiglitz type, the steady state level of marginal costs is given by the inverse of the mark-up of prices over marginal costs $\mu = \frac{\theta}{\theta-1}$. This can be easily checked for the model at hand by setting inflation equal to its steady state value Π and solving for marginal costs.

[38] See for instance Gali/Gertler (2000).

The steady state discount rate $\overline{\rho}$ can be replaced by $\beta\frac{u'(C_{t+1})}{u'(C_t)}$. Hence for the case of logarithmic utility ($u(C_t) = \ln(C_t)$) and given that consumption will grow at the same rate as output in the steady state, we have

$$\begin{aligned}\pi_t &= \frac{(\theta-1)}{\varphi}\widehat{mc}_t + \beta E_t \pi_{t+1} \qquad (27)\\ \pi_t &= \frac{\pi_{t-1}}{1+\beta} + \frac{(\theta-1)\widehat{mc}_t}{\varphi(1+\beta)} + \frac{\beta}{1+\beta}E_t\pi_{t+1}\end{aligned}$$

Note that these equations resemble very closely specifications which are obtained by Woodford (2003) under the assumption of Calvo contracts but different degrees of indexing of the prices of those firms which can not reoptimise prices to past inflation. While the first equation of (27) is a purely forward looking Phillips curve and corresponds to no indexing in the Calvo model, the second specification equation corresponds to full indexing among those firms which are not able to reoptimise their prices. In fact, for both equations, the coefficients on expected future inflation and the coefficient on lagged inflation in the second equation exactly match Woodfords results.[39] In the simulations carried out below, we will use the hybrid Phillips curve because of the generally superior empirical performance of Phillips Curves featuring lagged inflation. Furthermore, implies that disinflation is always costly in terms of output and employment because the weight on lagged inflation exceeds 0.5.[40] Costliness is a feature of real world disinflations, and recent test of the hybrid Phillips Curve by Jondeau/ Bihan (2005) suggests that the coefficients on past and expected inflation exceed 0.5 in France, Germany and the Euro area as a whole and are in fact quantitatively close to the values in equation (27) for standard values of β.[41] Furthermore, if disinflation were costless even in the short run, the persistent effects of monetary policy which are the subject of this paper could not arise.

As the simulation experiment which we aim to conduct is a disinflation, we have to introduce an inflationary shock, like for instance an oil price shock. We account for such a shock by adding a so called "cost-push shock" u_t to the Philips curve equation

$$\pi_t = \frac{\pi_{t-1}}{1+\beta} + \frac{(\theta-1)\widehat{mc}_t}{\varphi(1+\beta)} + \frac{\beta}{1+\beta}E_t\pi_{t+1} + u_t \qquad (28)$$

where u_t captures all factors affecting marginal costs which are not explicitly accounted for elsewhere in the model. While it would certainly be desirable to derive such a shock from first principles, like for instance explicitly including energy in the

[39] See Woodford (2003), p. 215.

[40] As was shown by Chadha et al (1992), this is a sufficient condition to prevent the path of disinflation from being completely costless. Intuitively, a reduction in expected inflation reduces inflation today, and a lower coefficient on expected inflation means that today's inflation will be reduced by less for any given output level. See Chadha et al (1992), p. 403.

[41] See Jondeau/ Bihan (2005), pp. 521-550.

production function, the road taken here has the advantage of simplicity and is also in line with the New-Keynesian literature.[42]

5.6 Monetary Policy

Monetary Policy will be assumed to follow a simple Taylor type nominal interest rate rule. The exact specification will vary across simulations, though all specifications will include a a lagged dependent variable in order to account for the interest rate inertia observed in the data. The baseline rule will be a rule which reacts to current inflation and the lagged output gap:

$$i_t = (1-\rho)\,\bar{i} + (1-\rho)\,\psi_\pi \pi_t + (1-\rho)\,\psi_Y gp_{t-1} + \rho i_{t-1} \tag{29}$$

where $\bar{i}$, ρ and gp_t denote the long-run real interest rate (recall that inflation is zero in the steady state), the degree of interest rate smoothing and the output gap, respectively, while ψ_π and ψ_Y denote the long run coefficients on inflation and the output gap. Hence the central bank responds to the lagged value of the output gap but current values of inflation, on the grounds that output data is usually available only with a lag while data on inflation arrives earlier.

The output gap is the percentage deviation from potential Y_t^n, which is the output level which would set marginal costs equal to its long run level μ^{-1}, given the capital stock, and the previous periods real wage. As can be obtained from equation (27), this would ensure that -in the long run- inflation is neither rising nor falling. The employment level corresponding to this output level will be referred to as "natural employment" n_t^n. Potential output is derived by substituting the equation for the rental on capital from (12) and the wage setting equation (15) into (16) and setting $mc_t = \mu^{-1}$. The natural levels of output and employment are then given by the values of Y_t^n and n_t^n solving

$$\mu^{-1} = \frac{(A_t)^{\frac{\alpha}{1-\alpha}} \left(\phi_1 n_t^n\right)^\alpha w_{t-1} \exp(a + b n_t^n)}{X_t K_t} \tag{30}$$

$$Y_t^n = A_t K_t (n_t^n \phi_1)^{1-\alpha} \tag{31}$$

Obviously both Y_t^n and n_t^n depend on K_t. More specifically a higher capital stock relative to past real wages w_{t-1} will allow for a higher natural employment level. This is due to the fact that a higher capital stock reduces marginal costs: an increase in K_t by 1% given employment will reduce marginal costs by one percent. Similarly, a fall of or just slower growth of K_t relative to wage growth will reduce the level of employment compatible with marginal costs.

[42] See for instance Clarida et al (1999), pages 1665 and 1667.

5.7 The aggregate equations

This section summarises the models aggregate equations developed above for convenience of the reader. As many of the economies variables are growing in the steady state ($Y_{t,} C_t, I_t, w_t, K_t$), simulation of the model requires normalising those variables in a way which produces constant steady state values. It is very convenient from a technical point of view to normalise with respect to the capital stock. Just how that is done is shown in the appendix, as well as how the steady state values of the variables are calculated.

5.7.1 Aggregate Demand

Aggregate demand consists of consumption and investment.

$$Y_t = C_t + I_t \tag{32}$$

We will assume logarithmic utility so that the consumption Euler equation becomes

$$\frac{1}{\beta} E_t \left[C_{t+1} \frac{1+\pi_{t+1}}{1+i_t} \right] = C_t \tag{33}$$

Using the functional form for the Investment goods production function suggested above,

$$I_t = K_t \left(\frac{1}{\gamma} \left(\frac{K_{t+1}}{K_t} \right)^{\gamma} - \left(\frac{1}{\gamma} - \delta \right) \right) \tag{34}$$

and using it on equations (23), K_{t+1} and the price of capital goods $\overline{Q}_t$ we have:

$$\begin{aligned} \frac{1}{1+i_t} E_t \left[(1+\pi_{t+1}) \left(\overline{Q}_{t+1} + r_{t+1}^k \right) \right] &= \left(\frac{K_{t+1}}{K_t} \right)^{\gamma-1} \\ \overline{Q_t} &= \left(\frac{K_{t+1}}{K_t} \right)^{\gamma} \left(1 - \frac{1}{\gamma} \right) + \frac{1}{\gamma} - \delta \end{aligned} \tag{35}$$

where r_t^k denotes the rental on capital. Capital stock growth will be positively affected by the capital rental in period t+1 r_{t+1}^k paid by the goods producing sector and the repurchase price $\overline{Q}_{t+1}$ paid by the investment goods sector, which in turn is determined by the demand for investment goods by the capital goods producing sector in t+1 to produce capital goods to be sold in t+2. The capital rental is determined by the first of equations (15).

$$r_t^k = \alpha mc_t A((n_t - \overline{n}) \phi_1)^{1-\alpha} \tag{36}$$

Note that future employment is thus positively to current capital growth because it increases the (future) marginal product of capital and, to the extend that the increase in employment is more sustained, the future demand for capital goods by the investment goods producing sector Q_t, which in turn increases investment in the current period. This, in turn will tend to reduce marginal costs via equation (19). These dynamics are at the heart of the simulations discussed later.

5.7.2 Aggregate supply

What follows are the equations for marginal costs, wage setting and employment. From (19) we have for marginal costs

$$mc_t = \frac{\left(r_t^k\right)^\alpha w_t^{1-\alpha}}{A\alpha^\alpha(1-\alpha)^{1-\alpha}(\phi_1 K_t)^{1-\alpha}} \tag{37}$$

where $X_t == A_t^{\frac{1}{1-\alpha}}(1-\alpha)\phi_1$. Wages are set according to equation (15):

$$\log w_t = a + b\left(n_t - \overline{n}\right) + \log w_{t-1} \tag{38}$$

Employment can be written as a function of output using the aggregate production function $Y_t = A_t K_t((n_t - \overline{n})\,\phi_1)^{1-\alpha}$ via

$$n_t = \frac{1}{\phi_1 A_t^{\frac{1}{1-\alpha}}}\left(\frac{Y_t}{K_t}\right)^{\frac{1}{1-\alpha}} + \overline{n} \tag{39}$$

while the evolution of prices is determined by the Phillips Curve (28)

$$\pi_t = \frac{\pi_{t-1}}{1+\beta} + \frac{(\theta-1)\widehat{mc}_t}{\varphi\,(1+\beta)} + \frac{\beta}{1+\beta}E_t\pi_{t+1} + u_t \tag{40}$$

where u_t is a cost push variable which is used to introduce the possibility of an inflationary shock (like an oil price shock). It will be instructive to solve this equation forward to get

$$\pi_t - \pi_{t-1} = \frac{\theta-1}{\varphi}\sum_{i=0}^{\infty}\left(E_t\widehat{mc}_{t+i}\right) + (1+\beta)\sum_{i=0}^{\infty}E_t u_{t+i} \tag{41}$$

[43] This equation is a forward looking version of the traditional accelerationist Phillips Curve and says that inflation will accelerate if the sum of current and expected future marginal costs and current and future shocks exceed zero.

Finally, policy is specified by equation (29)

$$i_t = (1-\rho)\,\overline{i} + (1-\rho)\,\psi_\pi\pi_t + (1-\rho)\,\psi_Y gp_{t-1} + \rho i_{t-1} \tag{42}$$

[43] The derivation is shown in the Appendix. For comparison see Woodford (2003), p. 215.

6 Calibration

The non-policy parameters of the baseline calibration can be obtained from table 1. A main purpose of the baseline calibration was to create steady state growth rates of output per capita and unemployment which are in the order of magnitude which could be found in big European economies like France, Germany or Italy in the seventies. This is the period about when unemployment started rising and the two oil crisis hit the Western World, which gave rise to the need to disinflate the economy. Therefore the calibration is such that in the steady state, productivity growth equals 1.06% per quarter and 4.3% per year and a steady state unemployment rate of 4%.

Some of the values deserve special mention beyond that. The value of the depreciation rate δ is obviously pretty high, as it implies an annual depreciation rate of 27.68%. This was necessary in order to keep the steady state growth rate within reasonable bounds. It might be justified by the fact that the source of growth in this model is human capital. The assumption that human capital is proportional to the capital stock means that it does not enter the model explicitly, which also means that it does not depreciate. This is certainly unrealistic, as the discovery of new ways of producing things will make some skills obsolete. Given that in this model, human capital is proportionate to the physical capital stock, increasing the depreciation rate could be seen as a way of accounting for the depreciation of human capital. Concerning γ which indexes the degree of adjustment costs, we use a baseline value of 10. This corresponds to an elasticity of adjustment costs to capital stock growth of about 9.9, which very much exceeds the value of 3 Woodford (2003) employs in his baseline calibration and is in the range assumed by Nolan/Thoenissen(2005).[44]

The slope of the wage growth function b will turn out to be crucial for the extent to which monetary policy affects the path of employment. At the same time the calibration of this parameter poses special difficulties because the specifications tested in the empirical literature are usually more complex than equation (18). We therefore estimated the equation for German data labour costs per hour and detrended logarithmised hours as a measure of employment using quarterly data ranging from 1975q1 to 2007q2 by two stage least squares to account for possible endogeneity of employment and using dummies. We think that hours provide a better representation of employment then the unemployment rate and its variations because it account for changes in overtime as well as for employment changes which are not reflected in the unemployment rate, like changes in over time or people leaving work which do not register as unemployed. We exclude the early 70s because during this period German unions were particularly militant and many observations for real wage growth are out of line with the rest of the sample, i.e. they are between 2 and 3,6% per quarter. The results are reported in the Appendix but are not very robust to the choice of the sample over which to estimate. Our estimate of b is 0.5 and is significant at the 10% level. In order to be on the safe side, however, we choose a value of 0.059.

[44]See Woodford (2004), p. 21, Woodford (2003), p. 368, and Nolan/ Thoenissen (2005), p.19.

The cost push shock u_t is set equal to 0.0055 for the first eight quarters and then drops to zero. To put it differently, we have a 0.55 percentage point increase in quarterly inflation given marginal costs, or an 2.22 percentage point increase at an annualised rate. This shock is still relatively modest compared to a real world oil crisis. Note that there is no endogenous persistence in the shock itself beyond the first four quarters, which implies that any persistence in the path of the variables and in particular employment beyond that point is endogenous.

The amount of overhead labour is then computed using equation (17) and amounts to 0.14, hence 14% of the labour force are employed as overhead workers.

Table 1: Baseline calibration of non-policy parameters

α	β	A	θ	δ	ϕ_1	φ	a	b	γ	u_{1-8}	$\overline{n}$	$\bar{i}$
1/3	0.99	0.55	10	0.063	0.452	63.26	-0.038	0.059	10	0.0055	0.14	0.021

The baseline calibration of the policy rule is taken from Clausen/ Meier (2003), who estimate a Bundesbank policy rule over the period from 1973 to 1998 using a real time measure of the output gap in order to account for the fact that the central banks information set does not include future levels of GDP, and thus the output gap estimate used in estimates of the policy rule should be based only on GDP levels known in the quarter when the decision on the interest rate is made.[45] Clausen/Meiers best performing procedure for estimating the output gap yields the values reported in table 2, which are all statistically significant, and in fact correspond to the original coefficients proposed by Taylor (1993) to characterise the policy of the Federal Reserve.[46] This is of particular interest for the coefficient on the output gap, because the Bundesbank was often perceived as paying much less attention to output than the Fed, which was also borne out by estimates of the Taylor rule.[47] Because this paper aims to explain long swings in Europe's big economies by the response of monetary policy to an inflationary shock, it seems a good test to use as baseline coefficients for the policy rule the least hawkish ones in the literature of Bundesbank Taylor rule estimates.

Table 2: Baseline calibration of the policy rule

ψ_π	ψ_Y	ρ
1.5	0.52/4	0.75

7 Simulation Results

We can now turn towards discussing the results of some simulations. The simulations have been conducted using the software Dynare which is a Matlab programme allowing for the simulation of deterministic models without requiring linearization. We do

[45]See Clausen/ Meier (2003), p. 2. Note that because Taylor rules are usually estimated using annualised inflation and interest rate data, the coefficient on the output gap has to be divided by 4 to adapt it to quarterly frequency.

[46]See Clausen/ Meier (2003), pp. 11-12 and p. 22.

[47]See for instance Clarida et. al (1998), p. 1045.

not linearize the model, with the exception of the Phillips Curve, because we expected the deviations from the steady state to be substantial in the other equations. The model code and additional results are available from the author upon request.[48]

In discussing the results we will focus on the dynamics of employment and the NAIRU, Inflation, marginal costs and the capital stock. Figure 13 plots actual employment (the diamond) and natural employment (the square) for quarter zero (the steady state) to quarter 110. Employment drops by a bit more than 15% on impact. Employment would be expected to decrease because the cost push shock will increase inflation which will ultimately lead to an increase in (ex ante) real interest rates via the policy rule (42), the path of which can be obtained from figure 15. As consumers and firms are forward looking, this causes a contraction of aggregate demand on impact even though the (ex ante) real interest rate actually drops at first because the central bank reacts to current rather than expected inflation. The size of the employment contraction is clearly extreme, but reflects the fact consumption and investment directly jump to the values consistent with utility and profit maximisation with no regard of their own past values. While the data generally shows investment to be indeed far more volatile than GDP, consumption is usually considerably less volatile, which conflicts with big jumps. This however, is a more general problem of models featuring purely forward looking households and its solution (potentially via changes to preferences like habit formation) is beyond the scope of this paper and is left for further study.

What becomes clear as well is that the shock as an extremely long lasting effects on employment. After about 14 quarters (8 quarters after the end of the shock), about two thirds of the on-impact loss in employment have vanished (see Table 3 for deviations of selected quarters) and employment is about 5% below its steady state value. However, from this point onwards, employment growth almost comes to a halt: quarterly increases are now in the order of magnitude of about 0.01 percentage points per quarter or less. This means that after a 100 quarters, or 25 years (23 years after the shock vanishes), employment is still about 4.43 percentage points below its steady state value, while after 200 Quarters (50 years) the difference is still about 3.67.

Table 3: Baseline -
percentage point Deviation of employment from its Steady State for selected Quarters

20	40	60	80	100	120	200	300
5	4.88	4.74	4.58	4.43	4.36	3.67	2.94

Given that unemployment thus remains at a high level, it is interesting to look at the evolution of inflation. An increase in unemployment above the NAIRU would be expected to cause disinflation. Figure 14 shows the evolution of inflation. Ss expected, inflation goes up following the shock, slowly gaining momentum through

[48] The programme and useful recourses on how to use it can be downloaded from http://www.cepremap.cnrs.fr/dynare/. See Juillard (1996) for description of the solution algorithm.

the presence of lagged inflation in the Phillips Curve, peaking in quarter 5 at 2.1% (about 8.4% at an annualised rate) and then starts falling. However, inflation stops falling in quarter 12 and then actually begins to increase, although unemployment is still high. A look at figure 17 which plots the deviation of marginal costs (the diamond) and the wage-capital ratio (the square), on whose evolution more will be said below, from their steady state values reveals the immediate reason for the halt of disinflation: after -as expected- dropping sharply in the first quarter as the reduction in employment depresses real wages via the wage setting function (38) and the capital rental r_t^k via (36), marginal cost returns to its steady state value after about 13 quarters. From equation (41) we know that $\widehat{mc}_t = 0$ today and in the future will indeed cause inflation to remain constant. Correspondingly, this is the time when actual employment reaches natural employment.

The recovery of marginal costs is driven by a drop in capital stock growth relative to real wage growth. Figure 18 shows that while real wage growth drops sharply, the growth rate of the capital stock falls by even more and remains considerably below real wage growth until 14 quarters after the beginning of the simulation. This will increase the wage-capital ratio, which will, as mentioned above, push up marginal costs (see the marginal cost equation (37)). Slower capital stock growth entails slower technological progress and thus slower growth of overall efficiency. Figure 17 confirms that it is the movement of real wages relative to the capital stock which drives marginal cost back up: Marginal costs and the wage-capital ratio move broadly in parallel.

Hence the recovery of actual employment has to slow down after about 14 quarters because employment arrives at the level beyond which any increase would lead to an acceleration of inflation because it would push real wage growth above the growth rate of the capital stock. To this the central bank would respond by raising interest rates. Figure 15 shows that the central bank stops lowering the real interest rate after 17 quarters, when it is about 0.3 percentage points below the steady state value. This is not very expansionary because the capital rental rate and the re-purchase price of capital goods are very much depressed as well. Figure 16 summarises this by plotting the present discounted value of an additional unit of capital, $\frac{1}{1+i_t} E_t \left[(1+\pi_{t+1}) \left(\overline{Q}_{t+1} + r_{t+1}^k \right) \right]$, also known as Tobin's Q, which is in fact the left hand side of the investment Euler equation in (35). Tobin's Q recovers quickly after the shock has passed but then reaches a plateau at a value of 1.072, which falls short of its steady state value by about 0.028. To put it differently, the real return on a unit of capital in excess of the risk free rate is 2.8% lower than before the shock, and the incentive to invest and thus capital stock growth is depressed accordingly. The reason why the central bank is not more expansionary in spite of unemployment still being high is that the output gap, which is really only a monotonous transformation of the deviation of marginal costs from its steady state value, is closed.

The speed of recovery is then governed by the relative growth rates of real wages and the capital stock. While it is not visible in figure 17, from quarter 17 onwards, the capital stock grows very slightly faster than real wages, and very slowly increases.

This causes a slow decline in the wage-capital ratio, as can be obtained from figure 16, and allows for a slow increase in employment because higher productivity growth implies firms can accommodate the increased real wage growth associated with a tighter labour market without facing an increase in marginal costs. This, in turn, again increases capital stock growth by increasing the marginal product of capital.

Thus the disinflation engineered by the central bank, while clearly successful, has come at a cost beyond a temporary reduction in employment: The unemployment level consistent with constant inflation, or $\widehat{mc}_t = 0$, has increased. Just as could be observed in Europe, a successful disinflation during which the economy goes into recession is followed by an increase in the NAIRU.

These results provoke the question how changes to the central banks reaction function affects the long-run paths of employment and inflation. Intuition would expect that a stronger weight on the output gap in the reaction function would lead to a smaller decrease in employment not just in the short but also in the long run because investment would be squeezed less, implying a more benign evolution of capital stock growth which would accommodate a smaller initial rise in the wage-capital ratio and higher employment levels. Therefore we increase the coefficient on the output gap, ψ_Y, to 1.5, leaving all other parameters the same. The corresponding evolution of employment can be obtained from figure 22. Indeed employment not only decreases by considerably less in the short run, but a difference of about 1 percentage point persists, as can be obtained from table 4, up until 300 quarters. Hence a less hawkish monetary policy has indeed very long-lasting benign effects on employment. Figure 21 shows that, as expected the wage-capital ratio, increases by less.

Table 4: $\psi_Y = 1.5$ - percentage point deviation of employment from its Steady State for selected Quarters

20	40	60	80	100	120	200	300
3.92	3.77	3.62	3.47	3.32	3.18	2.62	2.01

This, however, comes at the cost of a considerably stronger inflation surge during the lifetime of the cost-push shock. While in the baseline simulation, inflation peaks a (quarterly) rate of 2.1%, it now increases as high as 4.67%, implying an annual rate of 20%, and remains there for two quarters, as can be obtained from figure 20. Note however that inflation falls below its steady state value of zero in quarter 12, which is about two quarters later than in the baseline simulation. Thus the stronger acceleration in inflation is a short run phenomenon, while the gain in employment is of more long-run nature. At the same time it is true that inflation is pushed down a bit further below zero in case of a more hawkish central bank. Whether this is desirable or not would require a welfare analysis which is beyond the scope of this paper. However, it is illustrative to summarise the tradeoffs policymakers are facing by continuing to vary the output gap coefficient and to plot the resulting average annualised inflation rates against the corresponding average unemployment and natural unemployment rates. This done in figure 23 for values of ψ_Y between 0.3 and 3. Both curves are clearly downward sloping. As with traditional Phillips Curves, both curves become steeper as unemployment becomes lower. The unemployment Phillips Curve is always flatter than the NAIRU-Phillips Curve because monetary policy affects the path of actual unemployment in the short run more strongly than the NAIRU: its slope varies from -0.44 to -0.21 as unemployment increases while the slope of the NAIRU Philips curve varies from -2.23 to -0.93. Over the range of policy rules considered here, a 2.26 percentage point reduction in the average NAIRU is associated with a 3.02 increase in inflation. Hence we have, very much contracting conventional wisdom, a trade-off between inflation and unemployment over an extended period of time.

We will next consider how these results change if real wages are more flexible. Intuition would suggest that more flexible wages would cause a less persistent response to unemployment, because any given increase in unemployment leads to a stronger reduction in real wage growth than before, which will mitigate the drop of the capital stock growth rate relative to the real wage growth rate. Hence more employment can be accommodated without triggering an acceleration of inflation. Higher employment in turn raises the marginal product of capital, triggering more investment, pushing up capital stock growth and creating further room for employment expansion.

To investigate the quantitative implications of these mechanisms, we increase the slope of the real wage growth function b to 0.065, which corresponds to a change of a bit more than 10%. Figure 24 shows that both natural and actual employment recover more quickly than in the baseline case. The minimum of natural employment with more flexible real wage growth is about 1.7% above the minimum in the baseline simulation, which gives more room for an immediate non-inflationary recovery. Note also that the continuing recovery after employment has hit natural employment is more than twice as fast: employment recovers at a speed of about 0.02 to 0.03 percentage points per quarter. As a result, after 100 quarters, as can be obtained from table 5 the deviation from the steady state is only 1.21 percentage points as opposed to 4.43 percent in the baseline case. Still the degree of persistence observed here still exceeds by far what would commonly arise from more conventional New Keynesian models. However, viewing this and the previous results in conjunction clearly lend support to the view that as suggested by Blanchard, it is both "shocks and institutions" which are at the heart of explaining the evolution of unemployment, to the extent that institutions affect real wage flexibility.

Table 5: $b = 0.065$ **- percentage point Deviation of Employment from its Steady State for selected Quarters**

20	40	60	80	100	120	200	300
3.09	2.49	1.97	1.55	1.21	0.94	0.34	0.09

As a final robustness experiment, we change the slope of the adjustment cost function by increasing γ. It is clear and can also be verified from equation (38) that with adjustment costs rising faster in investment, investment will be less affected by changes in the present value of an additional unit of capital and thus will fall less in response to increases in the real interest rate or reductions in the capital rental, which in turn implies that reductions in employment will have a smaller effect on capital accumulation as well. All this would be expected to more benign path of employment both in the short as well as in the long run.

Increasing γ from 10 to 12 generates the employment path displayed in figure 26. As expected, both actual and natural employment decrease by less than in the baseline case and recover more quickly. Table 6 again reports the deviation of n for selected quarters.

Table 6: $\gamma = 12$ - Percentage point Deviation of Employment from its Steady State for selected Quarters

20	40	60	80	100	120	200	300
3.04	2.74	2.42	2.13	1.87	1.64	0.94	0.46

8 Conclusion

This paper is a first pass at overcoming the traditional separation between the short and the long run in modern macroeconomics by integrating a New Growth production technology featuring learning by doing which enters the economy as an externality into a New Keynesian model with unemployment. In doing so, it shows that a temporary, two year inflationary shock ("cost push shock"), combined with an interest rate rule of the central bank with standard coefficient values which lead the central bank to engineer a disinflation, can cause substantial and very persistent effects on unemployment. Under the baseline calibration, unemployment will be about 4.4 percentage points above its pre-shock value after about 120 quarters, or 30 years. At the same, inflation stops declining soon after the cost push shock has vanished. Thus the increase in unemployment represents an increase in the NAIRU.

The increase in the NAIRU is brought about by the decline in investment during the recession required to disinflate the economy. The capital stock, in this endogenous growth economy, has a much stronger effect on marginal costs than in models with a neoclassical production function. Thus, although wage growth declines as employment contracts, marginal cost returns back to its steady state level soon after the shock has vanished, which stops disinflation. The subsequent recovery is very slow because the central bank has no reason to lower interest rates. Its reaction function dictates, that it reacts only to inflation, which is constant, and the output gap, defined as the deviation of output from the level consistent with constant inflation, which is zero.

The model also shows that the central bank faces a trade-off between preventing a strong acceleration of inflation and quickly bringing inflation back to target on the one hand and preventing a persistent increase in unemployment on the other. A higher coefficient on the output gap has substantial and lasting benign effects on the path of employment. We also show that varying the output gap coefficient and plotting the resulting average unemployment rates and NAIRUs against the associated average inflation rates creates a downward sloping Phillips Curves.

Thus the model can contribute to explaining the evolution of European unemployment during the 1980s and beyond. Rising NAIRUs would be the consequence of the disinflations engineered as a response to the inflationary shocks of the late 1970s. It would also explain why, as found by Ball, countries which disinflated less and pursued a more expansionary monetary policy once the economy was on the disinflationary track like the United States experienced smaller or no increases in the NAIRU. The model also contributes to explaining the productivity slowdown.

At the same time, institutions still matter to the extent that they affect the response of real wage growth to the level of unemployment: Higher real wage flexibility considerably decreases the persistence of the increase in unemployment following the shock. Thus the paper clearly lends support to the view that, as suggested by Blanchard, it is both "shocks and institutions" which are at the heart of explaining the evolution of unemployment.

An obvious extension of the analysis presented here would be to introduce a government and non Ricardian consumers to allow for expansionary effects of debt-financed government expenditure. While disinflation was somewhat less an issue in Europe during the 1990s than during the 80s, the "road towards Maastricht" forced those EU countries aiming to adopt the Euro in 1998 to pursue an austere fiscal policy which entailed both reducing budget deficits and the public debt-GDP ratio. By contrast, the Reagan administration hugely increased public debt. While this policy is commonly accepted to have affected employment in the short run, it would be interesting to analyse their potential long run effects within a suitably modified version of the model proposed here.

9 Appendix A - Forward Solution of the Phillips curve

The Hybrid Phillips Curve of this model is

$$\pi_t = \frac{\pi_{t-1}}{1+\beta} + \frac{(\theta-1)\widehat{mc}_t}{\varphi(1+\beta)} + \frac{\beta}{1+\beta} E_t \pi_{t+1} + u_t$$

This can be rearranged to get

$$\pi_t - \pi_{t-1} = \frac{(\theta-1)\widehat{mc}_t}{\varphi} + (1+\beta)\, u_t + \beta \left(E_t \pi_{t+1} - \pi_t\right)$$

Defining $\pi_t - \pi_{t-1} \equiv S_t$, we have a forward looking first order difference equation. Using the forward operator F, which is defined such that $FX_t = X_{t+1}$ we can write

$$(1-\beta F) S_t = \frac{(\theta-1)\widehat{mc}_t}{\varphi} + (1+\beta)\, u_t \text{[49]}$$

[49] See Leslie (1993), pp.94-95.

Using the fact that $\frac{X_t}{1-\lambda F} = \sum_{i=0}^{\infty} \left(\lambda^i X_{t+i}\right)$ if $\lambda < 1$, we arrive at

$$\pi_t - \pi_{t-1} = \frac{\theta - 1}{\varphi} \sum_{i=0}^{\infty} (\widehat{mc}_{t+i}) + (1+\beta) \sum_{i=0}^{\infty} u_{t+i}$$

10 Appendix B - Normalised Version of the Model

The capital stock can be replaced by its growth rate while C_t, Y_t, I_t and w_t are normalised by the capital stock to enable the computation of steady state values for all variables. This is done in this appendix. The resulting equations are those which have been simulated.

10.1 Aggregate demand

$Y_t = C_t + I_t$, $\frac{Y_t}{K_t} \equiv F_t$, $\frac{C_t}{K_t} \equiv D_t$, $\frac{I_t}{K_t} \equiv R_t$, $\frac{K_{t+1}}{K_t} = 1 + g^k_{t+1}$

$$F_t = D_t + R_t \tag{43}$$

Consumption

$$\frac{1}{\beta} E_t \left[D_{t+1} \frac{1+\pi_{t+1}}{1+i_t} \right] \left(1 + g^k_{t+1}\right) = D_t \tag{44}$$

Investment

$$R_t = \frac{1}{\gamma} \left(1 + g^k_{t+1}\right)^{\gamma} - \left(\frac{1}{\gamma} - \delta\right) \tag{45}$$

$$\frac{1}{1+i_t} E_t \left[\frac{P_{t+1}}{P_t} (\overline{Q}_{t+1} + r^k_{t+1}) \right] = \left(1 + g^K_{t+1}\right)^{\gamma - 1} \tag{46}$$

$$\overline{Q_t} = \left(1 + g^K_{t+1}\right)^{\gamma} \left(1 - \frac{1}{\gamma}\right) + \frac{1}{\gamma} - \delta \tag{47}$$

The rental on capital:

$$r^k_t = \alpha mc_t A_t ((n_t - \overline{n})\, \phi_1)^{1-\alpha} \tag{48}$$

10.2 Aggregate supply

Substituting (15) into (40) gives

$$mc_t = \frac{F_t^{\frac{\alpha}{1-\alpha}} H_t}{X_t} \tag{49}$$

where $X == A^{\frac{1}{1-\alpha}}(1-\alpha)\phi_1$.

Wage Setting: $\ln w_t = \ln w_{t-1} + b\left(n_t - \overline{n}\right)$can be rewritten as $\ln H_t = a + b\left(n_t - \overline{n}\right) + \left(\frac{w_{t-1}}{K_{t-1}\left(1+g_t^k\right)}\right) = a + b\left(n_t - \overline{n}\right) + \ln\left(\frac{H_{t-1}}{\left(1+g_t^k\right)}\right)$

$$H_t = \exp(a + b\left(n_t - \overline{n}\right))\frac{H_{t-1}}{\left(1+g_t^k\right)} \tag{50}$$

or, in case of a linear function

$$H_t = \exp(a + b\left(n_t - \overline{n}\right))\frac{H_{t-1}}{\left(1+g_t^k\right)}$$

Employment: from $Y_t = A_t K_t((n_t - \overline{n})\,\phi_1)^{1-\alpha}$, we have

$$n_t = \frac{1}{\phi_1 A_t^{\frac{1}{1-\alpha}}} F_t^{\frac{1}{1-\alpha}} + \overline{n} \tag{51}$$

The Phillips Curve and the Policy rule do not contain any trended variables and therefore does not need to be normalised,

Natural output in the two equations determining natural employment and natural output has to be normalised as well. F_t^n, and "natural" employment nn_t,

$$\begin{aligned} \mu^{-1} &= \frac{\left(F_t^n\right)^{\frac{\alpha}{1-\alpha}} \exp(a + b\left(nn_t - \overline{n}\right))\frac{H_{t-1}}{\left(1+g_t^k\right)}}{X_t} \\ F_t^n &= A_t(n_t^n \phi_1)^{1-\alpha} \end{aligned} \tag{52}$$

given last periods wage/ capital ratio H_{t-1} and this periods capital stock growth rate rate g_t^k (which was also determined in the t-1 by the then investment decision). As can be obtained from the equations, both nF_t and natural employment can change over time. In particular, an increase in g_t^k will increase natural employment and nF_t , as it is no possible for firms to accommodate stronger real wage increases. The output gap gp_t is then calculated as

$$gp_t = \frac{Y_t - Y_t^n}{Y_t^n}\left(\frac{K_t}{K_t}\right) = \frac{F_t - F_t^n}{F_t^n} \tag{53}$$

11 Appendix C: Steady State relations

This Appendix shows how to calculate the steady state values for the system developed in Appendix B. To so we will first derive a steady state relation between the level of employment and the steady state growth rate.

In the steady state, it has to be true that consumption, the capital stock and output all have to grow at the same rate g, to be determined subsequently. This is also true for the real risk less rate. Seting $D_t = D_{t+1}$ in (47) gives

$$\frac{P_t}{P_{t+1}}[1+i_t] = \frac{1+g}{\beta} \tag{54}$$

To eliminate the real risk less rate, we turn to the capital producing sector. In the steady state, the capital rental r_t^k and the repurchase price of capital $\overline{Q_t}$ will be the same in each period. With the capital stock growing at rate g rather than staying constant, we have

$$\frac{1}{1+i_t}E_t\left[\frac{P_{t+1}}{P_t}(\overline{Q}+r^k)\right] = (1+g)^{\gamma-1} \tag{55}$$

$$\overline{Q} = (1+g)^{\gamma}\left(1-\frac{1}{\gamma}\right)+\frac{1}{\gamma}-\delta \tag{56}$$

Furthermore, it is possible to express the rental rate on capital r_t^k as a function of employment alone by using the steady state relation $mc_t = \mu^{-1}$, which yields

$$r^k = \alpha\mu^{-1}A((n-\overline{n})\,\phi_1)^{1-\alpha} \tag{57}$$

Substituting the second into the first equation of (58) for $\overline{Q}_t$ and substituting for r^k yield

$$\left[(1+g)^{\gamma}\left(1-\frac{1}{\gamma}\right)+\frac{1}{\gamma}-\delta+\alpha\mu^{-1}A(n\phi_1)^{1-\alpha}\right] = (1+g)^{\gamma-1}(1+i_t)\frac{P_t}{P_{t+1}} \tag{58}$$

Substituting (26) and rearranging then yield an equation containing only the growth rate of the economy and employment:

$$(1+g)^{\gamma}(1-\frac{1}{\gamma})-\frac{1+g}{\beta}+\frac{1}{\gamma}-\delta+\alpha\mu^{-1}A(n\phi_1)^{1-\alpha}=0 \tag{59}$$

This equation can be solved for g,which yields

$$g = \left(\frac{\frac{1}{\gamma}+\frac{J}{1-\alpha}n^{1-\alpha}-\delta}{-1+\frac{1}{\gamma}+\frac{1}{\beta}}\right)^{1/\gamma}-1 \tag{60}$$

Differentiating yields

$$\frac{\partial g}{\partial n_t} = \frac{1}{\gamma}\left(\frac{\frac{1}{\gamma}+\frac{J}{1-\alpha}n^{1-\alpha}-\delta}{-1+\frac{1}{\gamma}+\frac{1}{\beta}}\right)^{1/\gamma-1}\frac{Jn^{-\alpha}}{-1+\frac{1}{\gamma}+\frac{1}{\beta}}>0$$

This is the steady state growth rate which is borne out by the marginal product of capital in the endogenous growth economy. It is easily verified that it is concave in employment.

It is straightforward to show that the real wage implied by the desired mark-up grows at the same rate as output and the capital stock by using $mc_t = \mu^{-1}$ on (40). This yields

$$\begin{aligned} w_t &= K_t \phi_1 \left(\frac{\mu^{-1} A \alpha^{\alpha} (1-\alpha)^{1-\alpha}}{(r^k)^{\alpha}} \right)^{1/(1-\alpha)} && (61) \\ \Delta \ln w_t &= \Delta \ln K_t = g && (62) \end{aligned}$$

Hence in the steady state, the real wage has to grow at the same rate as the capital stock. This means that equation (62) is in effect the dynamic, endogenous growth version of the familiar macroeconomic textbook price setting function: It gives the real wage growth rate compatible with marginal costs remaining constant and at its long run level. Unlike the textbook price setting function, this real wage growth rate is not constant but increases in employment: A higher steady state employment level implies a higher marginal product of capital, which triggers higher investment and thus faster capital stock- and thus productivity growth. Accordingly, the steady state levels of employment an the growth rate are determined by the intersection of (62) with the wage setting function (41).

Having determined g and $\overline{n}$, the determination of the steady state values of $F_t, D_t, R_t H_t, r_t^k$ and i_t is now straightforward. For F we have

$$F = A((n_t - \overline{n})\,\phi_1)^{1-\alpha} \tag{63}$$

from the production function. For R_t, we have from the investment good production function (48)

$$R = \frac{1}{\gamma}(1+g)^{\gamma} - \left(\frac{1}{\gamma} - \delta\right) \tag{64}$$

D can then be determined as a residual via

$$D = F - R \tag{65}$$

H is computed using the cost-minimisation FOC for labour

$$H = (1-\alpha)\mu^{-1}\frac{F}{n} \tag{66}$$

r^k is computed via (51)

$$r^k = \alpha\mu^{-1} A((n - \overline{n})\,\phi_1)^{1-\alpha} \tag{67}$$

The steady state value of i_t is computed by setting $P_t = P_{t+1}$ in (57), as inflation equals zero in the steady state which yields

$$\bar{i} = \frac{1+g}{\beta} - 1$$

Note that this is also the intercept of the interest rate rule of the central bank.

12 Appendix C: Estimation of the Wage Setting Function

We estimate the real wage growth function using German data ranging from 1970Q1 to 2007Q2. Our dataset includes Western German data up to 1991Q4 and from then on data for the unified country. All data is taken from a publication of the German "Statistisches Bundesamt".[50] As a measure of employment, we use detrended log hours. The detrending is necessary to account for population growth and changes in participation rates. Following Danthine/Kurman (2004) we use a linear time[51] trend. We account for reunification by using both an intercept dummy equalling 0 from 1970q1 to 1991q4 and 1 afterwards, and a slope dummy in 1992q1 as well. As a measure for labour costs, we the "Arbeitnehmerentgeld" per hour worked, which includes the full tax wedge.

We then estimate $\Delta \log \text{real}\, wage_t = c + \log hours_t + d92Q1$, where $d92Q1$ denotes an intercept dummy equalling one in 1992Q1 and zero everywhere else. The later is again to account for reunification. We tried a slope dummy as well but it was not significant. We use two stage least squares to account for the possible endogeneity of employment. As instruments, we choose $\Delta \log \text{real}\, wage_{t-1}, \log hours_{t-1}$(following again Danthine/Kurman (2004))$, c$ and $d92Q1$.[52]

As was already mentioned above, we exclude the period from 1970q1 to 1974Q4 because this was a time of extreme union militancy. During this period, there are five observations for real wage growth exceeding 2%, which are much higher than in the remainder of the sample. Fit (as measured by the adjusted R^2) strongly improves when we exclude those observations, and the result also becomes more efficient as the standard error strongly decreases. Note that we use Newey-West Standard Errors because the Breusch-Godfrey LM test for serial correlation rejects the hypotheses of no serial correlation at the 5% level. The result is reported in Table 7. For comparison, we report the results we obtain when we include 1970Q1 to 1974Q4 in Table 8.

Table 7
Dependent Variable: WG
Method: Two-Stage Least Squares
Date: 10/08/07 Time: 19:57
Sample: 1975Q1 2007Q2
Included observations: 130
Newey-West HAC Standard Errors & Covariance (lag truncation=4)
Instrument list: WG(-1) C N(-1) D92Q1

[50] See Statistisches Bundesamt (2006) and Statistisches Bundesamt (2007).
[51] See Danthine/ Kurman (2004), p. 139 to 140.
[52] See Danthine/ Kurman (2004), p. 121.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004598	0.000620	7.412018	0.0000
N	0.049527	0.026158	1.893348	0.0606
D92Q1	-0.110605	0.001401	-78.96046	0.0000

R-squared	0.515379	Mean dependent var	0.003571
Adjusted R-squared	0.507747	S.D. dependent var	0.013283
S.E. of regression	0.009320	Sum squared resid	0.011030
F-statistic	68.20684	Durbin-Watson stat	2.368047
Prob(F-statistic)	0.000000		

Table 8
Dependent Variable: WG
Method: Two-Stage Least Squares
Date: 10/08/07 Time: 21:41
Sample (adjusted): 1970Q3 2007Q2
Included observations: 148 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=4)
Instrument list: WG(-1) C N(-1) D92Q1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.005652	0.000761	7.424010	0.0000
N	0.117692	0.042299	2.782361	0.0061
D92Q1	-0.113903	0.001816	-62.73856	0.0000

R-squared	0.470467	Mean dependent var	0.004856
Adjusted R-squared	0.463163	S.D. dependent var	0.013383
S.E. of regression	0.009806	Sum squared resid	0.013942
F-statistic	66.51135	Durbin-Watson stat	2.287624
Prob(F-statistic)	0.000000		

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13 Figures

Figure 7: Western Germany - NAWRU and Trend Productivity

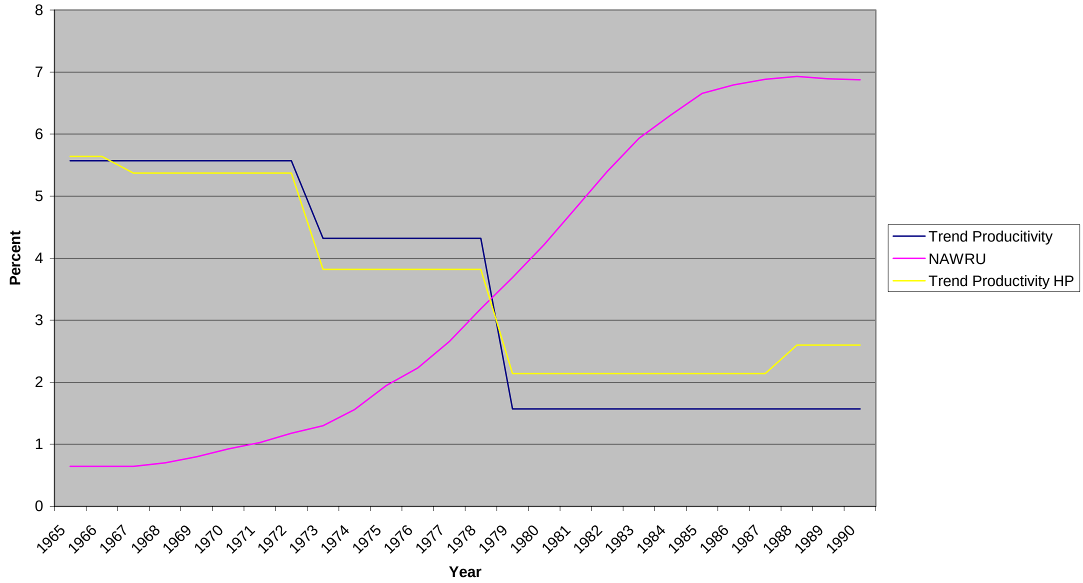


Figure 8: Spain - Trend Productivity and NAWRU

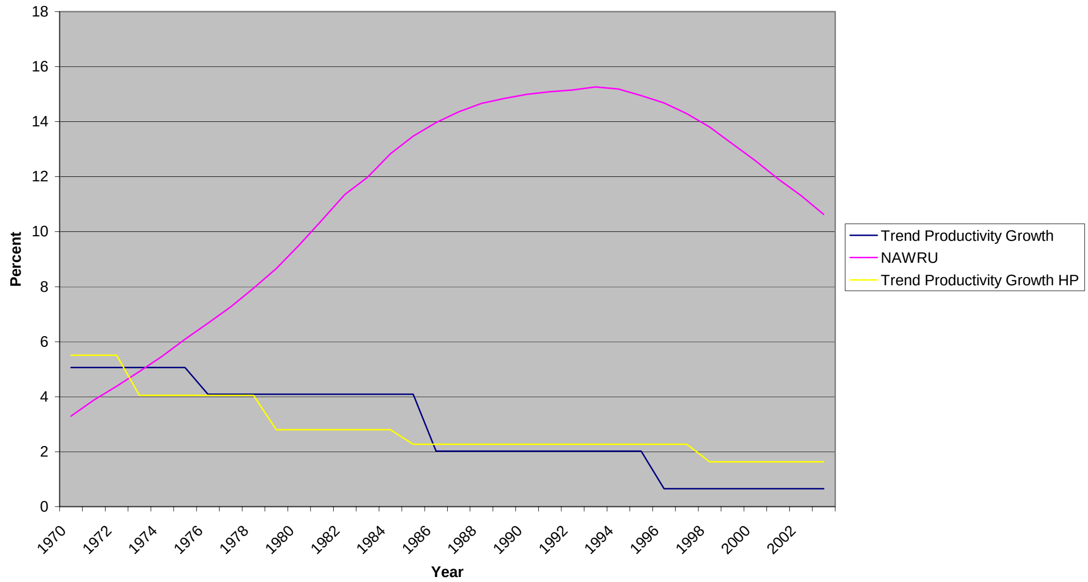


Figure 9: France Trend Productivity and NAWRU

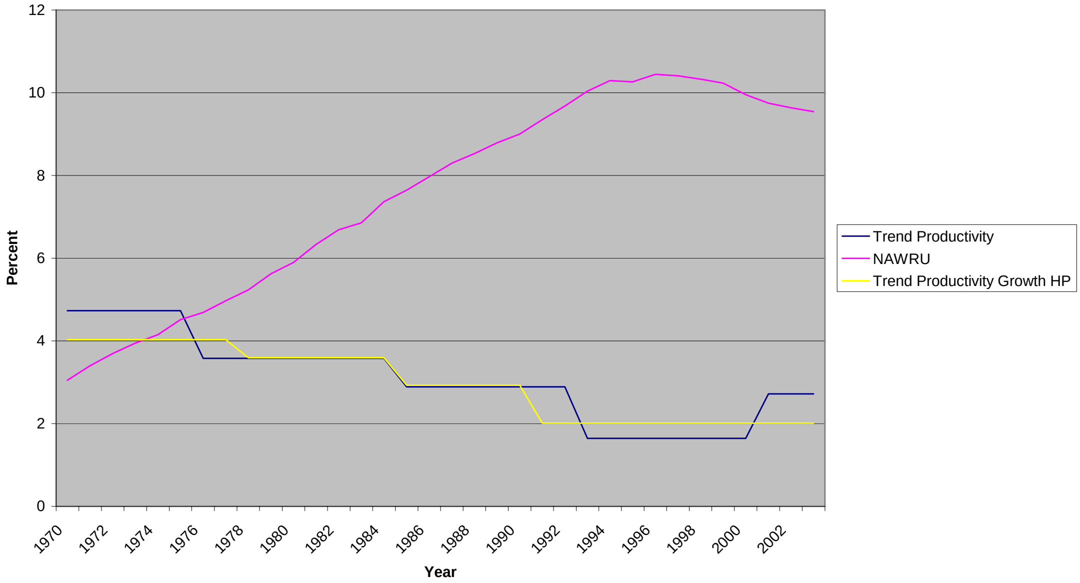


Figure 10: Italy - Trend Productivity and NAWRU

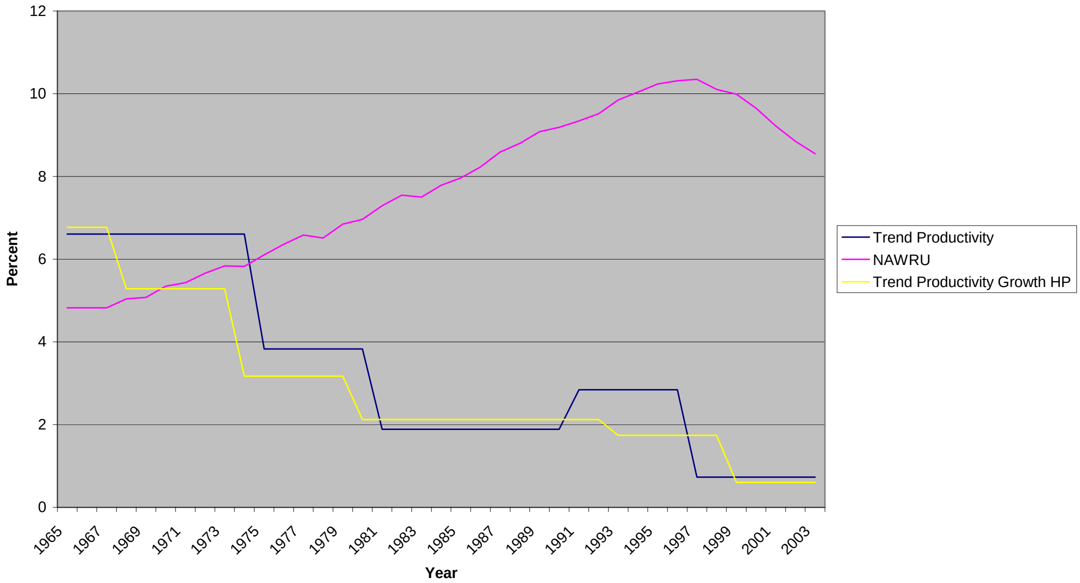


Figure 11 United States - NAWRU and Trend Productivity

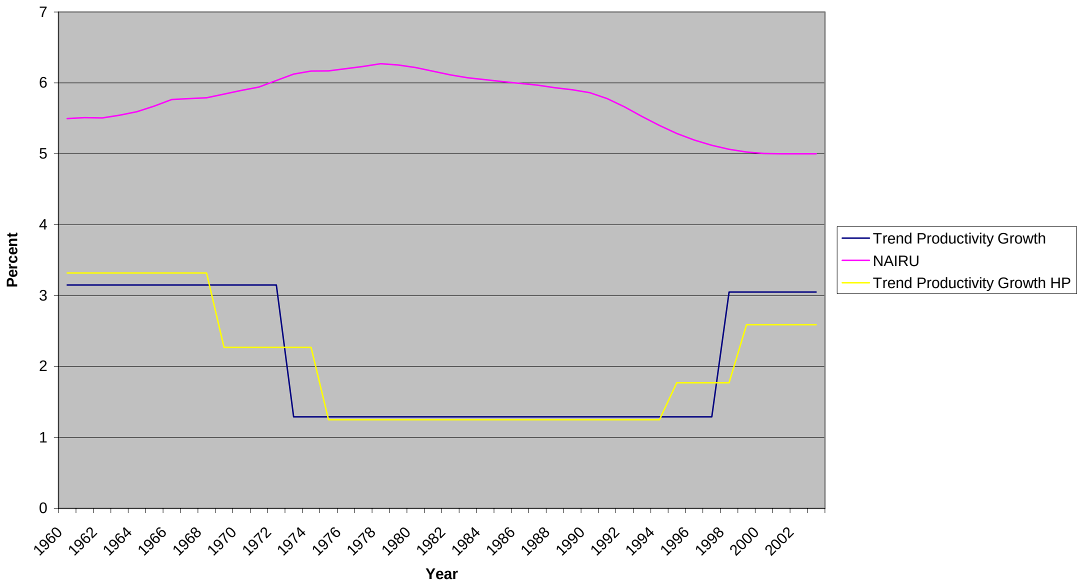


Figure 12 - United Kingdom - NAWRU and Trend Productivity

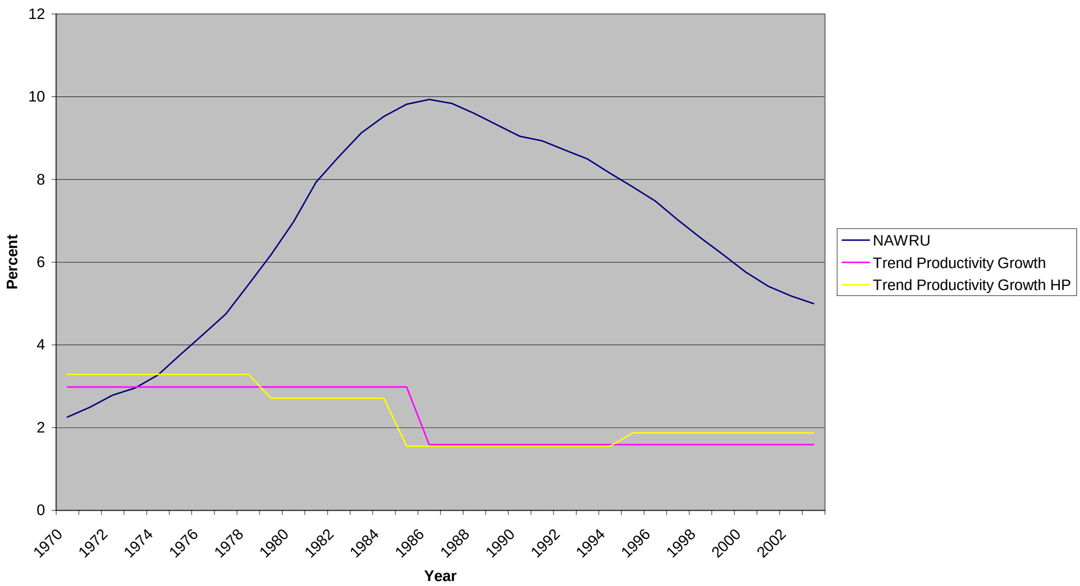


Figure 13: Baseline - Employment and natural Employment

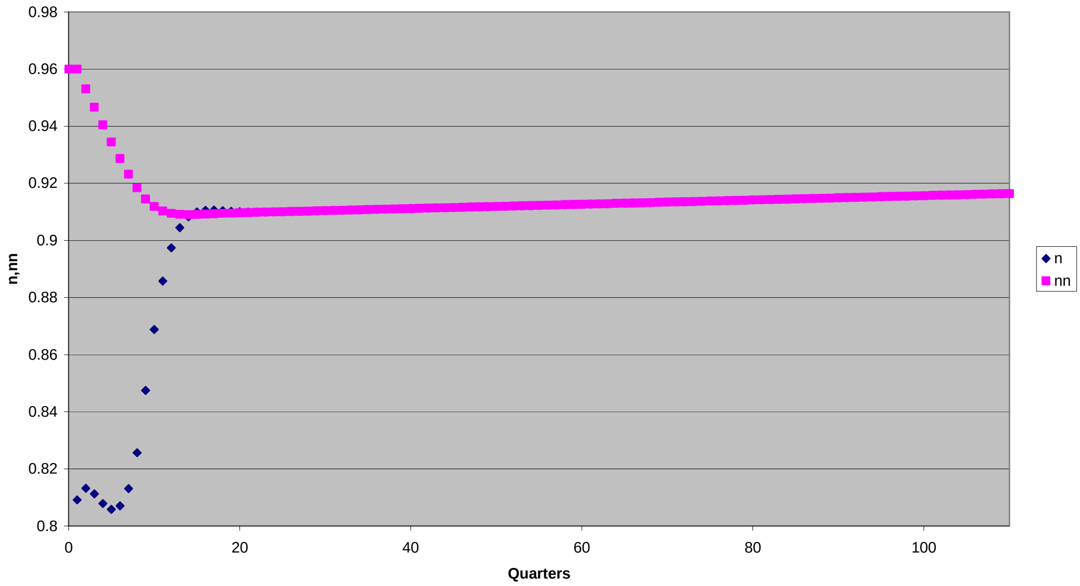


Figure 14: Inflation - Baseline

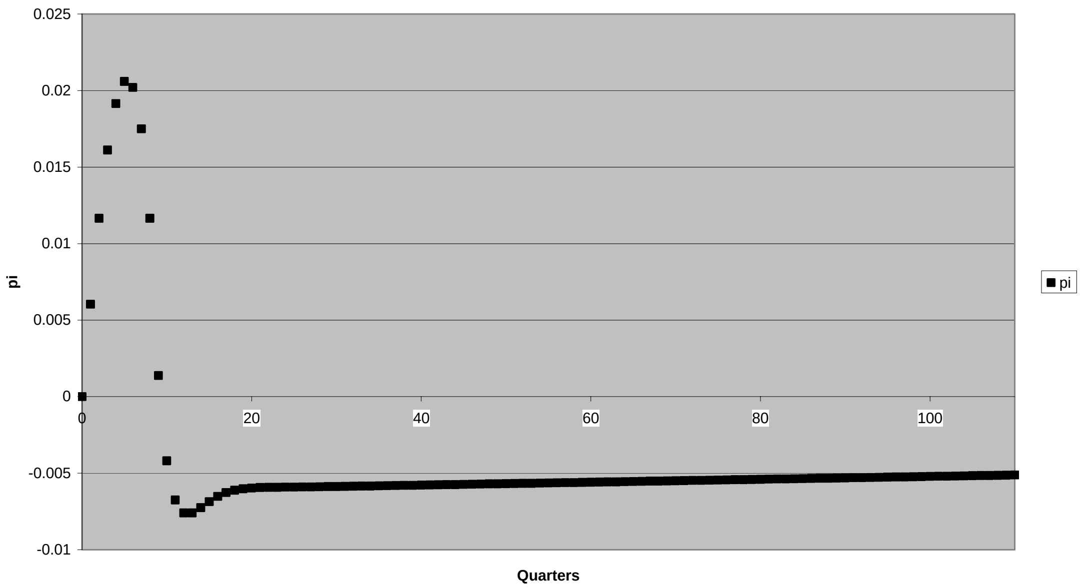


Figure 15: Baseline - Real Interest Rate

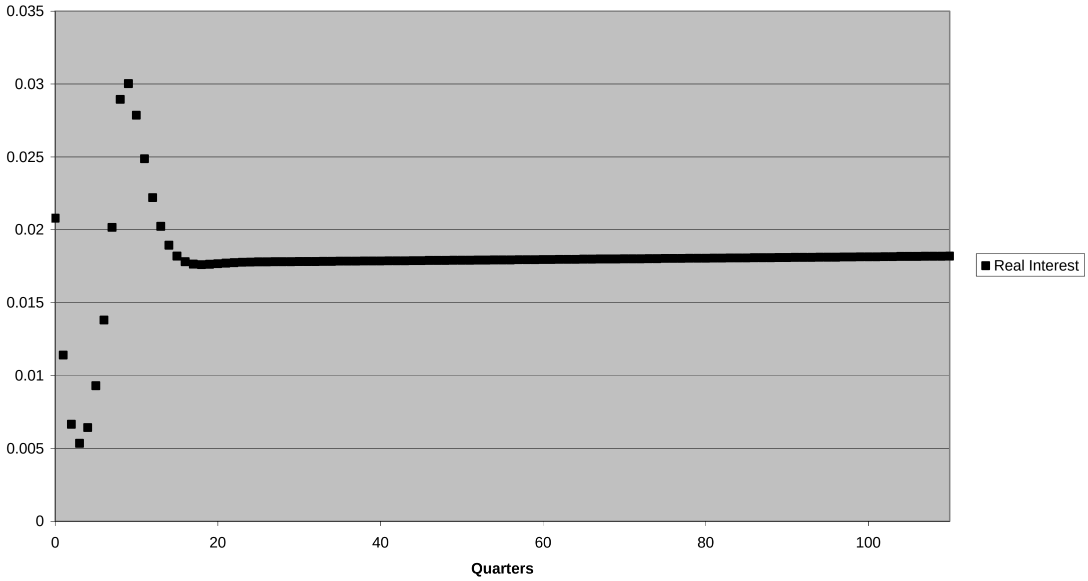


Figure 16: Baseline - Tobin's Q

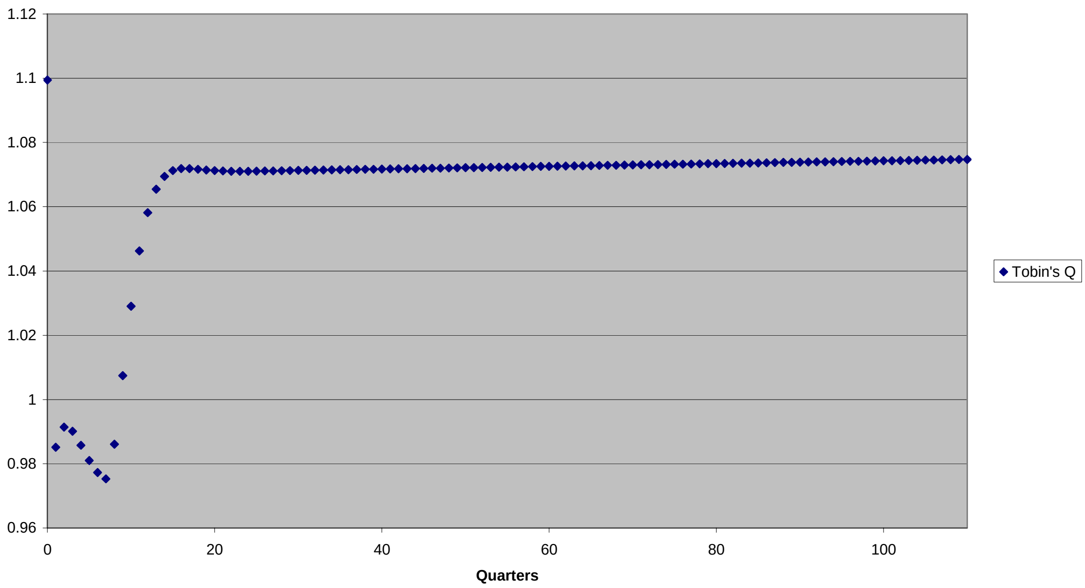


Figure 17: Baseline - Marginal Costs and Wage-Capital Ratio

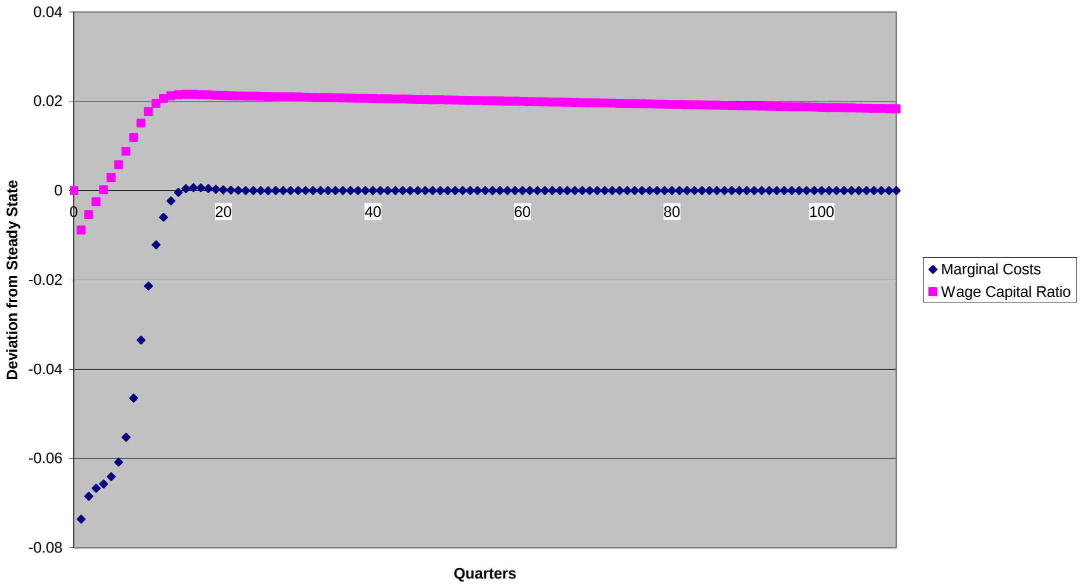


Figure 18: Baseline - Capital Stock and Wage Growth

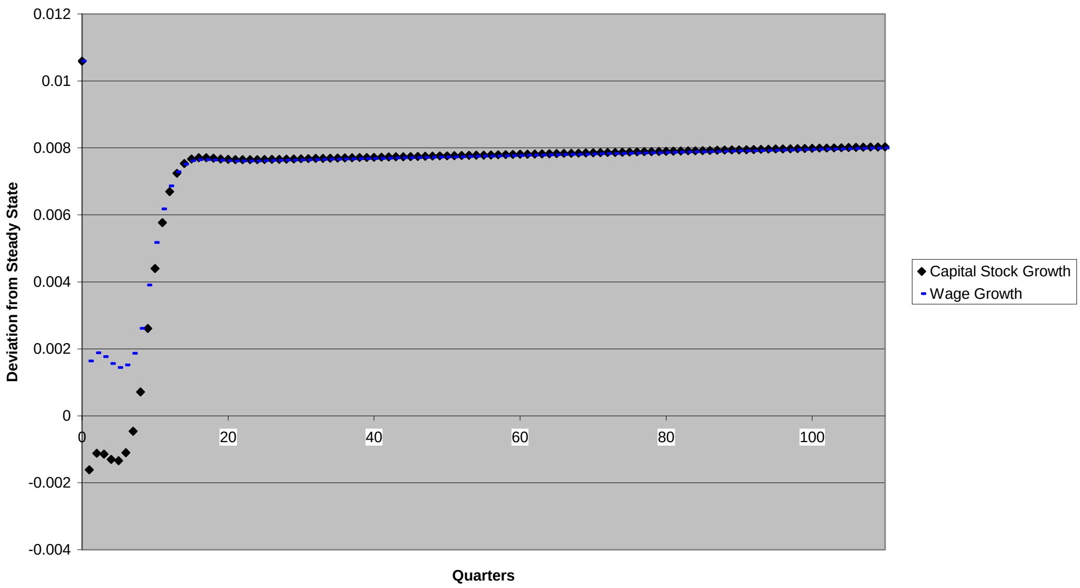


Figure 20: f=1.5 - Inflation

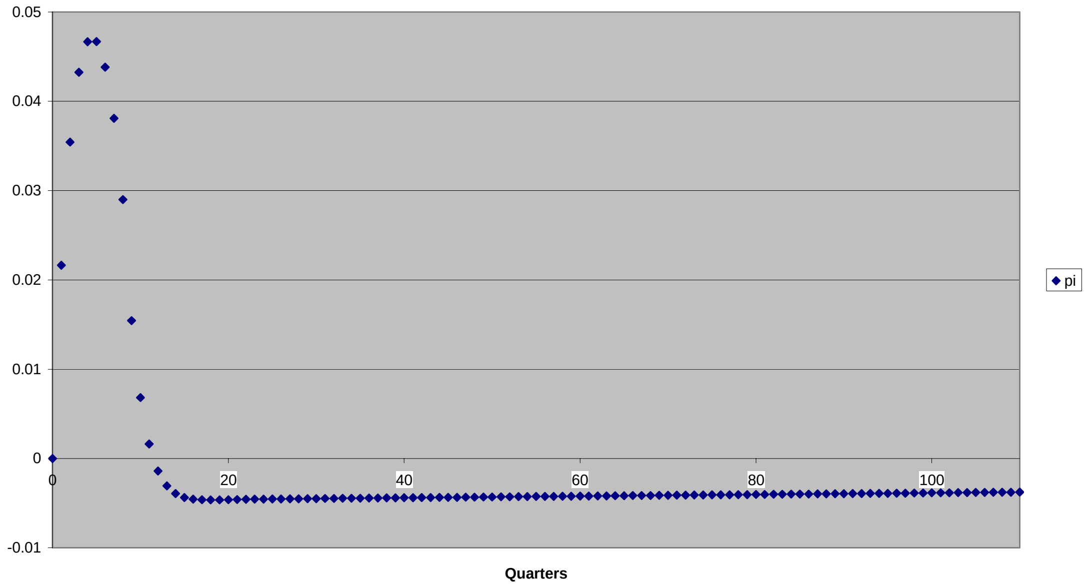


Figure 21: f=1.5 - Marginal Costs and Wage-Capital Ratio

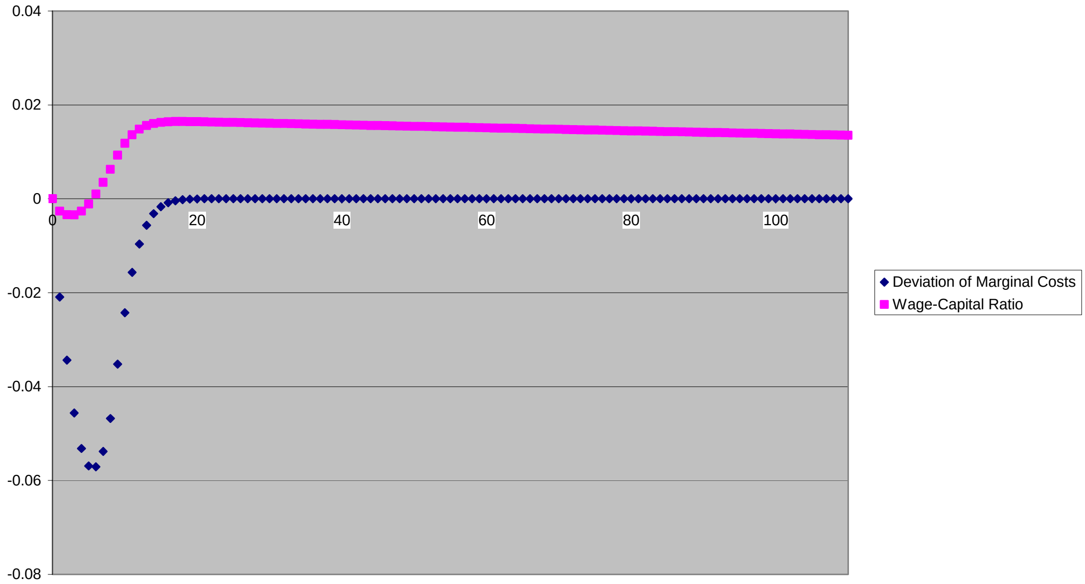


Figure 22: f=1.5 - Employment and Natural Employment

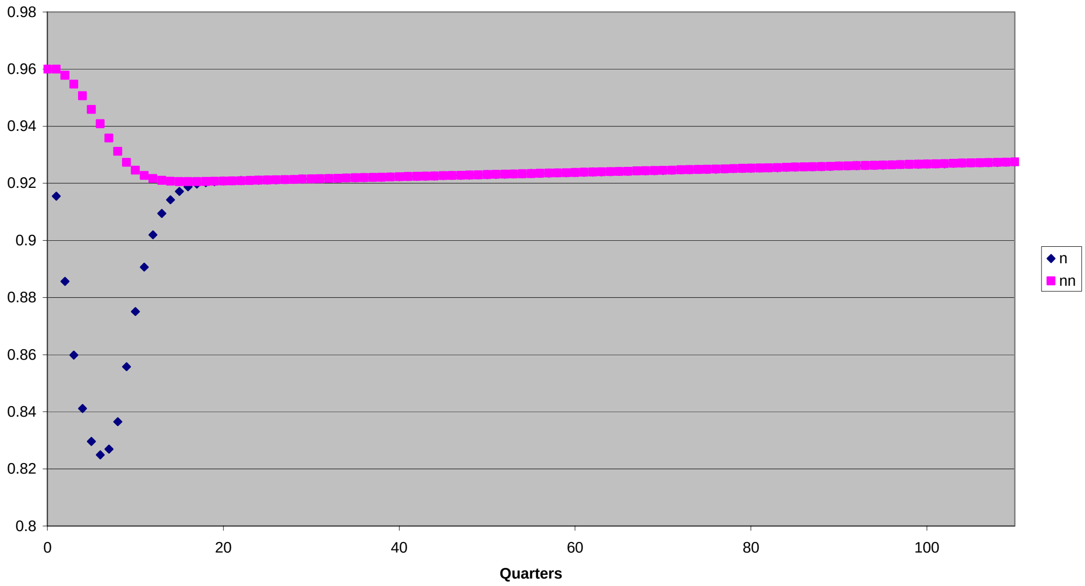
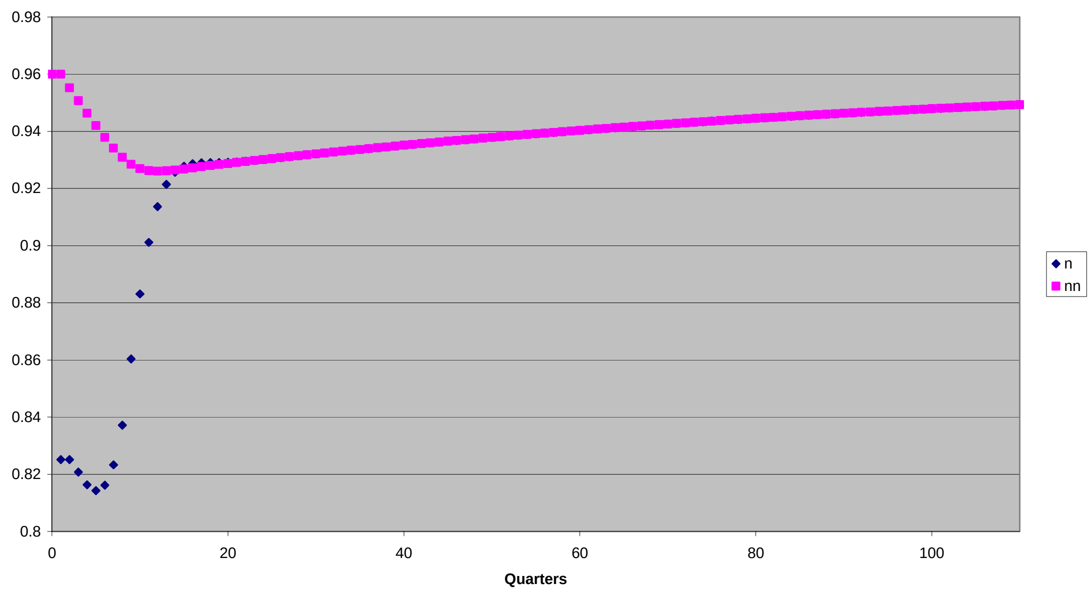


Figure 23: Inflation (annualised) and Unemployment for f=0.3 to f=3

Figure 24: b=0.065 - Employment and Natural Employment



b=0.065 - Figure 25: Marginal Costs and Wage Capital Ratio

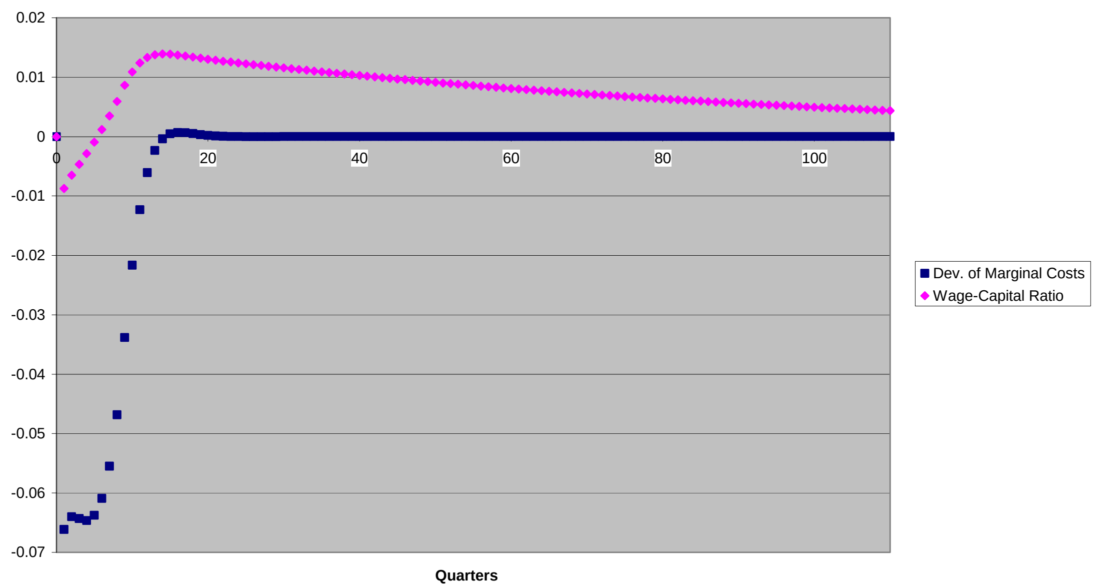


Figure 26: Gamma=12 - Employment and natural Employment