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Credit-Driven Investment, Heterogeneous Labour Markets and Macroeconomic Dynamics

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Keywords: Macroeconomic (In-)Stability, Segmented Labor Markets, Business Cycles, Fiscal and Monetary Policy Rules

JEL Codes: E12, E24, E31, E52

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1 Introduction

One of the most widely accepted stylized facts in macroeconomics is the existence of different types of frictions in the labor markets such as geographical and skill mismatches between vacancies and unemployed workers, labor search costs and the existence of asymmetric and/or incomplete information among the market participants. Following the seminal contributions by Diamond (1982), Mortensen (1982) and Pissarides (1985), a large body of literature based on the search-theoretic framework has investigated the role such types of frictions for the short- and long-run dynamics of unemployment, see also Pissarides (2000).

A particularity of models in this modeling approach is that the structural or natural unemployment therein arises solely due the existence of such frictions, and not from a persistently low level of aggregate demand, as a more Keynesian perspective would consider. Accordingly, the implicit policy recommendation of this type of models is the reduction of such frictions, either through direct action or indirect incentives to the labor market participants, and not the implementation of more expansive fiscal and monetary policies. This view is however not uncontested: in many non-neoclassical frameworks, long-term unemployment can be influenced by current economic conditions, as the wide literature around the endogenous NAIRU discussion has shown. A particularly interesting contribution which brings together the search theoretic and the Keynesian approaches within an stochastic multi-sectoral approach is discussed in Aoki and Yoshikawa (2011). There, a special focus is put on the modelling of workers' heterogeneity through the use of unemployed pools, which are clusters of workers of a similar type, and the use of the concept of ultrametric distance to model, among other things, the probability that a worker laid off in a sector i may be hired in a sector j. In this context unemployment movements are derived from frictions in the labor market, but Aoki and Yoshikawa also demonstrate that – through Okun's law– a large fraction of cyclical unemployment is due to demand and output fluctuations.

The main aim of our paper is also to investigate employment and unemployment movements in heterogeneous labor markets. We in particular refer to two segments of the labor markets where employment is given by contractual agreements of with different time horizon. Yet, to match the empirically observable aggregate fluctuations credit driven investment is introduced. We thus highlight the importance of the credit channel in a full-fledged model of the real sector with Keynesian quantity adjustment processes and a wage and price Phillips curve, representing what is usually called the dynamics of the supply-side. We have a simple form of Okun's law as link between the goods and the labor market and the loan rate, firms' indebtedness and (indirectly) the state of liquidity of the economy as arguments in the investment function. The theory of price inflation is of an advanced Tobinian type (Tobin , 1975) and augmented by the Chiarella and Flaschel (2000) concept of an inflation climate which provides inertia to the process of goods price inflation. The loan rate governing the economy is set by the asset holders, while the supply of new loans is generated endogenously and follows the loan demand of firms. The model exhibits at least six important feedback channels, two on the side of quantities, two nominal ones and two within the processes that generate and direct credit. This give rise to potentials for instability as well as stability.

Our framework is constructed in such way so that simplified, lower dimensional versions of the model can be obtained by setting parameters describing specific feedback effects from one sector to another equal to zero. Starting from a low dimensional sub-dynamics, we show stability of the full 7D model through a 'cascade of stable matrices' approach if the feedback chains are sufficiently tranquil in their transmission mechanisms. Yet, this variety of feedback channels will overthrow the stability of it by way of so-called Hopf-bifurcations if they become sufficiently strong, and will sooner or later generate cyclical explosiveness around the balanced growth path if not tamed by behavioral non-linearities far off the steady state.

The remainder of this paper is organized as follows. In section 2 we set up the baseline theoretical framework, deriving its unique steady state as well as its intensive form representation in section 3. We discuss the conditions for local stability of the system's steady state, local stability properties in section 4, and analyze the model's potentials for instability and cycles through numerical simulation in section 5. Section 6 draws some concluding remarks from this study.

2 The Baseline Framework

2.1 The Household Sector

The first module describes the consumption behavior of the household sector reflecting our two segments of the labor market.So there are worker households of type I and II. Moreover, there are pure asset holders (indexed by c) who also provide the flow savings into financial assets. Workers of type II do not save, due to the low income they are receiving relative to workers of type I. We do not allow for credit flows to type II of workers, the inclusion of which could be used to model aspects of the subprime crisis of 2008. Moreover, the savings of workers of type I are held as deposits in workers' banks with a 100% reserve ratio (as it is often assumed when no multiplier effects are considered). As there is no explicit system of unemployment benefits in this formulation of the household sector, we just assume that the government expenditures considered in the model do cover such benefits (through the term G_r in subsection 2.3).

We denote by $T_w^n = t_w^n K$ the taxes net of interest paid by the workers from their real income ωL^d – with $\omega = w/p$ representing the real wage – and by $T_c^n = t_c^n K$ the taxes net of interest paid by the capitalists on their real profits $\rho^{ge} K$, where ρ^{ge} is the expected gross rate of profit (including interest rate payments) defined as

$$\rho^{ge} = \frac{Y^e - \omega_1 L_1^d - \omega_2 L_2^d - r\Lambda/p - \delta K}{K}$$

where Y^e is the expected level of sales, ω_1 and ω_2 are the real wage for workers of type I and II, respectively, with L_1^d and L_2^d being the corresponding labor demands by the firms, r is the real loan interest rate and Λ is the firms' level of debt. We assume that workers of type I consume their disposable income with the propensity c_w (and save the remainder in the form of saving deposits), while capitalists consume with propensity c_c. Accordingly, total real private consumption is thus given by[1]

$$C = c_w(\omega_1 L_1^d + \alpha_2 \omega_1 L_2^d - t_w^n K) + c_c(\rho^{ge} K - t_c^n K), \quad \alpha_2 \in (0,1) \tag{1}$$

Compared to Chiarella and Flaschel (2000) and subsequent work, a novel feature of the present model is that the real loan rate is assumed to depend negatively on the output gap – measured as the deviation of the capacity utilization u (to be defined below) from its steady state level – (as screening becomes less costly in an economic boom), and positively both on inflation and on the nominal rate of interest paid on short-term bonds (with respect to which a Taylor rule will be formulated later on), that is

$$r = \bar{r} - r_u(u - u_o) + r_p \hat{p} + r_i(i - \bar{i}). \tag{2}$$

The other new feature of the current framework is that we consider two types of labor supplies, L_1 and L_2 given by

$$\dot{L}_1 = (\gamma_1^u e_1 - \gamma_1^d(1 - e_1))L_1 + \gamma L_1 \tag{3}$$
$$\dot{L}_2 = (\gamma_1^d(1 - e_1) - \gamma_1^u e_1)L_1 + \gamma L_2 \tag{4}$$

as they pertain to the segmented but interconnected two labor markets: the market for regular workers and the market for low income or atypical workers, respectively.

As Figure 1 illustrates, this interconnectedness is here modeled by the given portion γ^d of workers who leave the first labor market (in a downward direction 'd') into the second one (measured relative to the level of unemployed workers in the first labor market) and the portion γ^u of workers who are moving up in the income ladder by entering the first labor market out of the second one (again measured relative to the level of employed workers $e_1 L_1$ in the first labor market, e_1 being the employment rate). Of course, the opposite happens with respect to the labor supply on the second labor market. We have assumed that atypical or low-income workers receive a constant fraction α_2 of the wage for normal work and that they do not save, as a consequence of their low income streams.[2][3]

[1] We exclude the interest payments of the government as exercising a feedback on consumption by the assumed taxation rules (a standard assumption in macroeconomic theorizing) and reserve this feedback channel for a later extension of the model.

[2] It should be clear that if the two labor markets were not interconnected, the number of both types of workers would grow at an exogenous rate, given by the trend growth term in investment.

[3] At this point, the reader may also wonder what happens with the past savings of workers who were of type

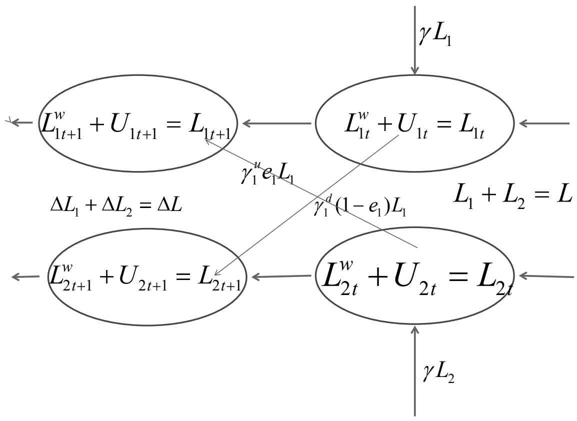

Figure 1: Segmented Labor Markets: Workforce and Unemployment Flows.

2.2 The Entrepreneurial Sector

We assume a production technology with fixed coefficients and denote by y^p = const. the output-capital ratio which determines the potential output $Y^p = y^p K$ of firms. The utilization rate of capital is denoted by $u = Y/Y^p = y/y^p$, where $y = Y/K$ is the actual output-capital ratio. Labor demand by firms is determined by output Y and average labor productivity x, the rate of growth of which we assume to be a given magnitude here, but in principle may be assumed to depend positively on both the infrastructure/capital stock ratio and on the labor share v, i.e.

$$L_i^d = Y/x_i,\ e_i = L_i^d/L_i,\ \hat{x} = \hat{x}_i = x(\mathcal{I}/K, v),\ i = 1, 2, \quad x' > 0 \tag{5}$$

Eq.(6) describes the change in the capital stock, captured by an investment function featuring several channels that influence demand for new capital outlays:

$$\hat{K} = i_\rho(\rho^{ge} - \rho_o^{ge}) - i_r(r - \hat{p} - \bar{r}) + i_o(u - u_o) - i_\lambda(\lambda - \lambda_o) + \gamma \tag{6}$$

Investment demand depends (i) positively on gross profits (and thus does not distinguish whether profits accrue to capital stock or credit owners); (ii) negatively on the the real loan rate (the real borrowing costs); (iii) negatively on the rate of utilization of capital, because of a Kaleckian retardation effect, and (iv) negatively on firms' debt to capital ratio λ. In addition, we consider investment demand as incorporating an exogenous trend term γ, which also determines all other trend terms considered in this model.

I and, due to any reason, become workers of type II (at least temporarily). Implicitly, we are just assuming here that workers of type I when they become workers of type II leave their past savings untouched as long as they work with a contract of type II (while interest is taken away from term through the assumed taxation rule by which – and because of the absence of any wealth effects – the influence of bonds in consumption is suppressed).

Changes in inventories, and thus the evolution of the firms' savings, are given by[4]

$$\dot{N}^i = S_f = Y_f = Y - Y^e. \tag{7}$$

Intended investment I is financed by net profits in the amount

$$\Pi_r^n = p(I + \dot{N} - \dot{N}^i) - \dot{\Lambda}^d, \tag{8}$$

as well as – and primarily – by new loans (via the implied Walras' law of flows which consolidate all aggregate of the budget equations), which are assumed to be fully supplied in the amount demanded by firms, i.e.

$$\dot{\Lambda}^d = (\alpha_f^o + \alpha_f^1(\lambda_o - \lambda))pI, \tag{9}$$

with windfall profits (or losses) $Y^d - Y^e$, defined as

$$Y^d - Y^e = C + I + G + \delta K - Y^e, \tag{10}$$

representing an additional third potential financing source.

2.3 The Government Sector

Government expenditures (which include unemployment benefits that are totally consumed) concern infrastructure investment G_i (keeping the ratio to the growing capital stock constant), the expenditure of the remaining taxes on social security and government consumption, denoted by G_r, and anti-cyclical fiscal policy consumption G_c, which is assumed here to be the only government expenditure type that is truly allowed to be debt financed (through the issue of risky assets, here perpetuities), with the government using debt to finance its additional expenditure in the bust, and reducing it in the boom. This anti-cyclical behavior is fluctuating around trend level of state expenditure γK. It is implicitly assumed that government cares about the unemployed in one form or another by means of its resources G_r.

$$\begin{aligned}
G_i &= \dot{R}^i + \delta R^i, \quad \dot{R}^i = \hat{K} R^i & (11)\\
G_c &= \gamma K - g_c(u - u_o)Y^e, & (12)\\
pG_c &= \dot{B} + p_p \dot{B}_p - iB_{cb} \to \dot{B}_p, \quad \dot{B} = \alpha_g p_p \dot{B}_p & (13)\\
pG_r &= T^n_{w+c} - (iB_{w+ca} + B_p + si^* B^*_{ca}) - pG_i = t^n_w pK + t^n_c pK - pG_i & (14)
\end{aligned}$$

We assume with respect to monetary policy a quite simple Taylor rule, according to which the monetary authorities simply raise the interest rate above a given level $\bar{i}$ if domestic prices are higher than their steady state value and vice versa, i.e.

$$i = \bar{i} + i_p(p - \bar{p}), \tag{15}$$

[4]We note the assumption that investment is not influenced by the state of Tobin's $q = p_k/p$, i.e., the asset markets, see e.g. ?, do not yet exercise an influence on the behavior of the real sector of the economy, which only depends on the loan rate setting of the banks run by the asset holders.

This fixing of the rate of interest i at each moment of time only demands that i is treated as given and that money supply M is endogenous and adjusted to the money demand of asset holders via open market operations in the form of short-term government bonds B. The reader may of course ask here why there is no goods-price inflation (or deflation) in the steady state of the model so that we can indeed use $\bar{p}$ as steady state price level in the above monetary policy rule. This will be implied by the assumed loan rate function as we will show in the next section.

2.4 Dynamics: Goods- and labor-market adjustment processes

Let us now discuss the modules of our theoretical framework that generate its laws of motions.

$$\begin{aligned}
Y^d &= C + I + \delta K + G, \quad G = G_i + G_c + G_r && (16)\\
N^d &= \alpha_{n^d} Y^e, \alpha_{n^d} > 0 && (17)\\
\dot{N}^i &= \gamma N^d + \beta_n (N^d - N), \beta_n > 0, && (18)\\
Y &= Y^e + \dot{N}^i, && (19)\\
\dot{Y}^e &= \gamma Y^e + \beta_{y^e}(Y^d - Y^e), && (20)\\
\dot{N} &= Y - Y^d = S - I, && (21)
\end{aligned}$$

Eq. (16) defines the aggregate demand function of the economy. The remaining equations represent the Metzlerian extension of the dynamic multiplier process, as introduced into models of the KMG variety in Chiarella and Flaschel (2000).

The wage-price dynamics is of the kind considered e.g. by Chiarella and Flaschel (2000) and Flaschel and Krolzig (2006), where $\hat{w}$ and $\hat{p}$ denote wage- and price inflation and where π^c represents the inflationary climate the economy is working in. Note that $\bar{e}_1$ is not the NAIRU rate of employment in this model, see the section on its steady state. We have two demand pressure terms on the labor first market, one inside and one outside of firms through the external labor market. Moreover, a better state in the second labor market leads to higher wage claims of the workers in the first labor market. Finally, we have a Blanchard and Katz (1999) type of error correction term which provides a moderating influence of wage share increases on the inflation rate of nominal wages.

$$\begin{aligned}
\hat{w}_1 &= \beta_{w1}(e_1 - \bar{e}_1) + \beta_{w2} e_2 + \beta_{wu}(u - u_o) - \beta_{wv}(v_1 - v_{1o}) + \kappa_w \hat{p}\\
&\quad + (1 - \kappa_w)\pi^c + \hat{x} && (22)\\
w &= (1 + \alpha_2) w_1, \quad \hat{w} = \hat{w}_1 = \hat{w}_2, \quad v_1 = w/p/x_1\\
\hat{p} &= \beta_{pu}(u - u_o) + \kappa_p(\hat{w} - \hat{x}) + (1 - \kappa_p)\pi^c && (23)\\
\dot{\pi}^c &= \beta_{\pi^c_c}(\hat{p} - \pi^c), && (24)
\end{aligned}$$

with $w = (1 + \alpha_2) w_1$, $\hat{w} = \hat{w}_1 = \hat{w}_2$, and $v_1 = w/p/x_1$.

The new equation in the wage-price dynamics is a two-level type of Okun's law

$$\hat{e}_i = \beta_e(u - \bar{u}) + \gamma - \hat{x} - \hat{L}_i \quad i = 1, 2 \tag{25}$$

which here links the utilization of the capital stock u (used as proxy for the utilization rate of the two types of workers employed by firms) and trend growth, $\beta_e(u-\bar{u})+\gamma$, to the recruitment rates on the external labor markets. These recruitment rates are however lowered through labor productivity growth $\hat{x}$ which depends positively on the infrastructure-capital ratio $\mathcal{I}/K$, which is a given magnitude in the present model. Okun's Law thus reflects absorption as well as dismissal effects in the recruitment policy of firms (wherefrom the growth rates of labor supply have to be deducted in order to arrive at the growth in employment rates.

The use of these two laws of motion allows to dismiss the law of motion for the state variable $l = L/K$ of the KMG models of Chiarella and Flaschel (2000) and represents a convenient short-cut that makes the representation of the dynamics similar to the original Goodwin (1967) representation of the growth cycle of capitalist economies.[5] Note moreover that the system is not a pure supply side model as the one of Goodwin (1967), but driven by a business cycle subsystem of the considered Keynes-Metzler type (which determines the variable u at each moment in time). Further, total labor supply is adjusting passively to the needs of firms as expressed through the trend term in their investment behavior. Finally, the term $\bar{e}_1$ in eq.(22) is not the steady state level of regular employment, but an exogenous, inflationary barrier on wage growth.

3 Intensive form and balanced growth path

3.1 Intensive form

The model developed above implies the following laws of motion for the state variables $y^e = Y^e/K$ (expected total sales as a ratio of capital stock), $\nu = N/K$ (inventories to capital ratio, i.e. the business cycle quantity adjustment process), and the debt to capital ratio $\lambda = \Lambda/pK$:

$$\dot{y}^e = \beta_{y^e}(y_t^d(\cdot) - y^e) - i(\,\cdot\,)y^e, \tag{26}$$

$$\dot{\nu} = y - y_t^d(\cdot) - [i(\,\cdot\,) + \gamma]\nu, \tag{27}$$

$$\dot{\lambda} = [\alpha_f^o + \alpha_f^1(\lambda_o - \lambda)](i(\,\cdot\,) + \gamma) - [i(\,\cdot\,) + \gamma + \hat{p}]\lambda \tag{28}$$

The laws of motion of the wage-price spiral, now reformulated for the open economy case and now also including firms' recruitment policies and the law of motion for the inflationary

[5] We implicitly use the variable L_1^w for the workforce of firms in order to define the rate of employment as $e_1 = L_1^w/L_1$, but avoid to use as state variable the rate of utilization of this workforce by L_1^d/L_1^w, an expression which is here proxied by the variable u.

climate, read on the intensive form level:

$$\begin{aligned}
\hat{v} &= \kappa[(1-\kappa_p)(\beta_{w1}(e_1-\bar{e}_1)+\beta_{w2}e_2-\beta_{wv}(v_1-v_{1o})) \\
&\quad +((1-\kappa_p)\beta_{wu}-(1-\kappa_w)\beta_{pu})(u-u_o)] && (29)\\
\hat{e}_1 &= \beta_e(u-\bar{u})-\hat{x}-(\gamma_1^u e_1-\gamma_1^d(1-e_1)) && (30)\\
\hat{e}_2 &= \beta_e(u-\bar{u})-\hat{x}-(\gamma_1^d(1-e_1)-\gamma_1^u e_1)\frac{e_2x_2}{e_1x_1} && (31)\\
\dot{\pi}^c &= \beta_{\pi^c}(\hat{p}-\pi^c) && (32)\\
\hat{p} &= \pi^c+\kappa[\beta_{pu}(u-u_o)+\kappa_p(\beta_{w1}(e_1-\bar{e}_1)+\beta_{w2}e_2+\beta_{wu}(u-u_o)] && (33)\\
&\quad -\kappa\beta_{wv}(v-v_o))]
\end{aligned}$$

The postulated wage-price dynamics contains an inflation accelerating mechanism of the type first discussed in Tobin (1975). This feedback chain between the inflation rate and the underlying inflationary climate provides an advanced view of the process of inflation in a two Phillips curve framework in which both wage and price dynamics are taken into account. The 2D wage-price dynamics, together with the 2D Metzlerian description of the business cycle dynamics, and the 2D Goodwin (1967)-style interaction between income distribution and employment form the 6D dynamics of the economy we are considering in this paper.

The static elements that are needed to complete the description of the intensive form of the model are:.

$$\begin{aligned}
y &= (1+\gamma\alpha_{n^d})y^e+\beta_n(\alpha_{n^d}y^e-\nu), \quad u=y/y^p && (34)\\
\rho^{ge} &= y^e-\delta-(1+\alpha_2\frac{x_1}{x_2})v_1y && (35)\\
r &= \bar{r}-r_y(u-u_o)+r_p\hat{p}-r_ii_p(p-\bar{p}) && (36)\\
y^d &= c_w\left(\left(1+\alpha_2\frac{x_1}{x_2}\right)v_1y-t_w^n\right)+c_c(\rho^{ge}-t_c^n)+t_w^n+t_c^n+\gamma-g_c(y/y^p-u_o) && (37)\\
&\quad +i_\rho(\rho^{ge}-\rho_o^{ge})-i_r(r-\hat{p}-\bar{r})+i_o(u-u_o)-i_\lambda(\lambda-\lambda_o)+\delta+\gamma && (38)
\end{aligned}$$

3.2 Balanced growth

The following set of equations describe the interior steady state —or balanced growth path— of the macrodynamics we study in this paper. Due to the postulated taxation rules, the financial side does not influence the balanced growth path.

$$\pi_o^c = 0 = \hat{p}_o \tag{39}$$
$$y_o = u_o y^p, u_o = \bar{u} + \hat{x}/\beta_e \tag{40}$$
$$y_o^e = y_o/(1 + \gamma\alpha_{n^d}) \tag{41}$$
$$\nu_o = \alpha_{n^d} y_o^e \tag{42}$$
$$r_o = \bar{r} \tag{43}$$
$$p_o = \bar{p} \tag{44}$$
$$\hat{K}_o = \gamma \tag{45}$$
$$\lambda_o = \alpha_{fo} \tag{46}$$
$$\rho_o^{ge} = y_o^e - \delta - (1+\alpha_2)\left(\frac{x_1}{x_2}\right) v_{1o} y_o \tag{47}$$
$$e_{1o} = \frac{\gamma^d}{\gamma^d + \gamma^u}, \quad e_{2o} = \frac{\beta_{w1}(\bar{e}_1 - e_{1o})}{\beta_{w2}} \tag{48}$$
$$v_{1o} = \frac{(1-c_c)(y_o^e - \delta - t_c^n) - (1-c_w)t_w^n - 2\gamma}{\left(c_w\left(1+\alpha_2\frac{x_1}{x_2}\right) - c_c\right) y_o} \tag{49}$$

The main state variables are (intensive-form) expected sales y^e and inventories ν, together with the Tobin (1975) inflation-accelerating mechanism in the inflation rate $\hat{p}$ and inflationary climate π^c, and finally the Goodwin (1967) distributive cycle in the labor share v and aggregate employment rate e. As it is clear after a combined look at equations (45), (40), the demand side only explains income distribution but not the trend growth rate of the economy which, in the sense of Keynes, is determined by exogenous animal spirits.[6] The steady state rate of employment of type-1 workers (48) balances flows into and flows out of regular employment, and thus compares directly with the flow vision of (un)employment found in Pissarides (2000). Atypical employment is instead determined residually, and therefore depends positively on the wage inflation barrier $\bar{e}_1$ while negatively on the regular employment rate.

Next, consider the share of wages of regular employees in equation (49). Higher tax rates on asset-holders and workers both reduce v_{1o} because asset taxes reduce profits and therefore investment on the one hand, while higher wage income taxes lower consumption. On the other hand, the effect of an increase in both the propensity to consume of asset-holders and that of workers is ambiguous, because it ultimately depends on parameter values. Typically, in demand-driven models the spending multiplier increases in marginal propensities to consume, but here we are not considering only aggregate demand but also income distribution.

[6] It is beyond the scope of this paper to provide theoretical grounds to γ, the animal spirits term, especially considering the role of (pure) uncertainty over the longer run in the future development of capitalism.

3.3 Feedback channels

The feedback channels of the dynamics are summarized in tables 1-4. To begin with we note that two real accelerator processes on the market for goods are given by the Keynes-Metzler quantity adjustment process and a level-oriented Harrodian multiplier-accelerator process. These accelerator processes are weak if the parameters β_n, i_u are sufficiently small. In particular the first accelerator process will play no role in the analysis this paper is focussed on.

The two real accelerator processes on the market for goods, the Keynes-Metzler and the Harrodian one have already been mentioned. We consider the inventory mechanism first. An increase in expected sales increases intended inventory by firms. Goods making up these inventories will be produced, so output will increase. Aggregate disposable income will increase as a result, and demand and expectations about sales will be revised upward. Concerning the Harrodian dynamics, an increase in the utilization of capital stock will push the non-trend component of investment demand (the accelerator). Hence, aggregate demand will rise and so will expected sales, thus reinforcing the upward trend in utilization. Table 1 below outlines these feedback chains.

Table 1: **Metzlerian and Harrodian accelerator mechanisms**

1. *Metzlerian inventory accelerator:*

$$\uparrow y^e \quad \stackrel{+}{\longmapsto} n^i \stackrel{+}{\longmapsto} y \stackrel{+}{\longmapsto} y^d \stackrel{+}{\longmapsto} \uparrow\uparrow y^e$$

2. *Harrodian investment accelerator:*

$$\uparrow u \quad \stackrel{+}{\longmapsto} i(\cdot,\cdot,\cdot,\cdot) \stackrel{+}{\longmapsto} y^d \stackrel{+}{\longmapsto} y^e \stackrel{+}{\longmapsto} y \stackrel{+}{\longmapsto} \uparrow\uparrow u$$

Next, we look at the dynamic effects on output and inflation of an increase in expected sales via the loan rate. If expected sales increase, the loan rate is pushed down through the increase in utilization as described in the loan rate setting equation. This reflects that screening processes of firms become cheaper when economic activity and the utilization of the capital stock increases. Then, following the decreases in the loan rate through this financial accelerator, investment demand will rise *ceteris paribus*, thus increasing aggregate demand, bringing about an upward revision of sales expectation, and ultimately increasing output. This is a positive feedback chain which may lead to screening cycles from this partial perspective. The analysis of this credit channel is related to the work of Bernanke and Gertler in particular, see Bernanke and Gertler (1995) for example. Similarly, the process of higher inflation lowers the loan rate in real terms, thus again increasing the non-trend component of investment demand. Consequently, aggregate demand will rise, and so will expected sales. Utilization will increase, and the inflationary pressure will be reinforced because of the combined effect of goods markets– and labor markets– dynamics. These two processes are summarized in Table 2.

Table 2: **Destabilizing Minsky- and Tobin-type loan-rate channels on the real and the nominal level of the dynamics**

3. Expected-sales/Loan-rate accelerator:

$$\uparrow y^e \quad \overset{-}{\longmapsto} r \overset{+}{\longmapsto} i(\cdot,\cdot,\cdot,\cdot) \overset{+}{\longmapsto} y^d \overset{+}{\longmapsto} y^e \overset{+}{\longmapsto} \uparrow\uparrow y$$

4. Inflation/Loan-rate accelerator ($r_p < 1$):

$$\hat{p}\uparrow \quad \overset{-}{\longmapsto} r - \hat{p} \overset{+}{\longmapsto} i(\cdot,\cdot,\cdot,\cdot) \overset{+}{\longmapsto} y^d \overset{+}{\longmapsto} y^e \overset{+}{\longmapsto} u \overset{+}{\longmapsto} \hat{p}\uparrow\uparrow \quad [\uparrow \pi^s \overset{+}{\longmapsto} \uparrow \hat{p}]$$

Next, we look at two different effect of changes in the price level. On the one hand, a lower aggregate price level decreases the interest rate set by the central bank (mirroring the conventional Keynes effect). Through the loan rate setting equation, the loan rate will fall, thus pushing up investment demand, aggregate demand, expected sales, and in turn utilization and thus provides a check to the price level decreases through both goods–market and labor–market pressures. By contrast, and on the other hand, a lower price level increases the amount of outstanding debt in real term, thus depressing investment, aggregate demand, and sales expectations, and ultimately resulting in lower utilization and further deflationary pressures. This is nothing but Irving Fisher's debt-deflation spiral. Table 3 summarizes the stabilizing Keynes-effect and the destabilizing Fisher-effect in our model of Keynesian macrodynamics. The latter effect is particular disastrous in the case of price deflation, since monetary policy is then incapable of stopping such spiral.

Finally, we consider below the interaction of workers and firms within the wage-price spiral (their conflict about income distribution) with either wage– or profit–led goods demand regimes, coupled with those repercussions between the labor market and the goods market which are at the heart of the distributive cycle. The case of a positive response of wage share growth on economic activity is denoted as a *labor market-led* wage-adjustment process, while the case of a negative reaction of the wage share growth to higher economic activity is denoted as *goods market-led*. As discussed e.g. by Proaño et al. (2011) the extent to which the wage adjustment process is labor-market or goods-market led, combined with the characteristics of the demand regime of the economy, determines whether real wage and wage-share adjustments have stabilizing or destabilizing effects. Wage adjustments will be stabilizing if the

Table 3: **Deflation, credit-driven Keynes effect and Fisher debt effect**

5. *'Keynes'-type stabilizing price-level effect:*

$$\downarrow p(\downarrow i) \xmapsto{-} r \xmapsto{+} i(\cdot,\cdot,\cdot,\cdot) \xmapsto{+} y^d \xmapsto{+} y^e \xmapsto{+} u \xmapsto{+} \uparrow\uparrow p$$

6. *Fisher debt-deflation amplifier:*

$$\downarrow p \quad \xmapsto{+} \lambda \xmapsto{-} i(\cdot,\cdot,\cdot,\cdot) \xmapsto{-} y^d \xmapsto{-} y^e \xmapsto{-} u \xmapsto{-} \downarrow\downarrow p$$

negative response of investment to changes in real wages outweighs the positive response of consumption, and if wages are more flexible to labor demand pressures than prices to goods market pressures (or both vice versa). Conversely, if investment reacts less than consumption to changes in real wages and wages are more flexible than prices, or both vice versa, then real wage adjustments will be destabilizing. These findings clearly show that empirical analysis is needed in this matter and may lead to different results depending on the country and the time period under consideration. These effects are summarized in table 4.

4 Local stability analysis

We first consider the local asymptotic stability properties of the resulting dynamical system, before investigate the likeliness of local explosiveness and the resulting for global stabilizers.

Proposition 1. *The interior steady state of the dynamical system is locally asymptotically stable if inventory adjustment is sufficiently sluggish (β_n small), i_u sufficiently small, r_y sufficiently small, i_r sufficiently small, i_λ sufficiently small, goods market adjustment sufficiently fast (β_{y^e} large enough), fiscal policy sufficiently active and the wage-price spiral sufficiently inactive (and the second labor market only weakly influential). The distinction wage-led vs. profit-led is not of real importance in this context. Fiscal and monetary policy can be used to stabilize the economy.*

Sketch of proof: We first set the parameters $\beta_e, \beta_n, \beta_{\pi^c}, i_\lambda$ equal to zero and consider the then resulting 3D dynamics in the state variables v, y^e, p in this order. As sign structure of the Jacobian of the dynamics at the steady state we get in this case:

$$J = \begin{pmatrix} \partial v/\partial v & \partial v/\partial y^e & \partial v/\partial p \\ \partial y^e/\partial v & \partial y^e/\partial y^e & \partial y^e/\partial p \\ \partial p/\partial v & \partial p/\partial y^e & \partial p/\partial p \end{pmatrix} = \begin{pmatrix} - & + & 0 \\ \pm & - & - \\ - & + & 0 \end{pmatrix}.$$

Table 4: **Wage-price spiral and income distribution**

1. *'Rose (1967)'-type stabilizing real-wage / aggregate demand interaction (profit-led $i(v_1)$ dominates $s(v_1)$, labor-market-led case $\hat{w}(u)$ dominates $\hat{p}(u)$):*

$$\uparrow v_1 \overset{-}{\longmapsto} Y^d \overset{+}{\longmapsto} Y^e \overset{+}{\longmapsto} Y \overset{+}{\longmapsto} u \overset{+}{\longmapsto} v_1, \ i.e: \downarrow v_1$$

2. *'Rose (1967)'-type stabilizing real-wage / aggregate demand interaction (wage-led $i(v_1)$ dom.ted by $s(v_1)$, goods-market-led case $\hat{w}(u)$ dom.ted by $\hat{p}(u)$):*

$$\uparrow v_1 \overset{+}{\longmapsto} Y^d \overset{+}{\longmapsto} Y^e \overset{+}{\longmapsto} Y \overset{+}{\longmapsto} u \overset{-}{\longmapsto} v_1, \ i.e: \downarrow v_1$$

3. *'Rose (1967)'-type destabilizing real-wage / aggregate demand interaction (profit-led $i(v_1)$ dominates $s(v_1)$, goods-market-led case $\hat{w}(u)$ dom.ted by $\hat{p}(u)$):*

$$\uparrow v_1 \overset{-}{\longmapsto} Y^d \overset{+}{\longmapsto} Y^e \overset{+}{\longmapsto} Y \overset{+}{\longmapsto} u \overset{-}{\longmapsto} v_1, \ i.e: \uparrow\uparrow v_1$$

4. *'Rose (1967)'-type destabilizing real-wage / aggregate demand interaction (wage-led $i(v_1)$ dom.ted by $s(v_1)$, labor-market-led case $\hat{w}(u)$ dominates $\hat{p}(u)$):*

$$\uparrow v_1 \overset{+}{\longmapsto} Y^d \overset{+}{\longmapsto} Y^e \overset{+}{\longmapsto} Y \overset{+}{\longmapsto} u \overset{+}{\longmapsto} v_1, \ i.e: \uparrow\uparrow v_1$$

This sign structure easily implies the validity of the Routh-Hurwitz conditions for local asymptotic stability if the term J_{12} is positive (or negative) and sufficiently close to zero. In this case the dynamics of the wage share is weakly labor market led (or weakly goods market led). The generally cumbersome last Routh-Hurwitz condition is then obviously fulfilled, since this ensures that the negative of the 3D determinant is dominated by the negative of the product of the trace and the principal minors of order 2.

We note that the sign of J_{21}, i.e., the question of whether the economy is wage led or profit-led does not matter in such a situation. Note moreover that the subdynamics in the state variables v, y^e represents a demand driven Goodwin (1967) growth cycle model, the distributive cycle of Barbosa-Filho and Taylor (2006) in fact, see also Proaño et al. (2011), with terms in the diagonal of the Jacobian being negative, i.e., the dual Keynesian forces are stabilizing the wage-led case if sufficiently pronounced. In addition the Keynes-effect in the investment function helps to improve the stability if the wage-led feedback effect becomes stronger.

Making the parameters β_e positive gives – after suitable row operations – for the determinant of the resulting 4D situation (in the order v, p, y^e, e) the qualitative expression:

$$|J| = \begin{vmatrix} 0 & 0 & 0 & + \\ + & 0 & 0 & 0 \\ 0 & - & 0 & 0 \\ 0 & 0 & + & 0 \end{vmatrix}.$$

This determinant is obviously positive. This implies that the 4th eigenvalue must be negative for small β_e, since the stability of the 3D system is not disturbed thereby (due to the continuous dependence of eigenvalues on the parameters of the dynamics).

Repeating the same for β_n (and the order v, p, y^e, e, ν) gives

$$|J| = \begin{vmatrix} 0 & 0 & 0 & + & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 \\ 0 & 0 & + & 0 & 0 \\ 0 & 0 & 0 & 0 & - \end{vmatrix}$$

which then obviously provides a negative determinant and thus again a negative eigenvalue (in place of a zero root) and thus the stability of the steady state if β_n is chosen sufficiently small. This simple result of only a negative entry in the diagonal of the resulting enlarged determinant also holds in the 6D case when β_{π^e} becomes positive and also when i_λ becomes positive and the full 7D system is considered. This completes the proof of the proposition.

The reader should note that the results on the alternating signs of the determinants of the 4D, 5D, 6D and 7D cases hold independently of the stability conditions for the 3D case and are thus completely general in nature. Using the law of motion for the price level p, the last row of the full 7D dynamics can be reduced to $(0, 0, 0, 0, 0, 0, -)$ by row operations which do not change the sign of the full 7D determinant which therefore must be negative.

At this point it should be stated that this proposition shows that convergence towards balanced growth is not what can be expected from the Keynesian macrodynamics considered in this paper. Indeed, even though we were able to show the resulting 6D subsystem is locally asymptotically stable under certain conditions revealed through the application of the cascades of matrices approach (see in particular Chiarella et al. (2006)), such characteristics of the system should not be considered as necessarily given. Further, it should be clear to the reader that the true extent of such stability considerations can only be investigated by means of numerical methods, since global analytical methods are not available here. Nonetheless, the importance of the proposition is that it helps when simulating the dynamics, since one can look by its help for basins of asymptotic stability first, and then start analyzing the dynamic behavior of the system when such conditions are relaxed.[7]

[7] Of course, global boundedness and thus the persistency of persistent oscillations can be further ensured by adding appropriate behavioral nonlinearities to the model as in Chiarella and Flaschel (2000) and later related work.

5 Numerical simulations of the model

This section illustrates some of the properties of the model with numerical simulation. Numerical simulations were produced using E&F Chaos software, which was developed at the Center for Nonlinear Dynamics and Finance (CeNDEF) at the Faculty of Economics and Business, Universiteit van Amsterdam, see Diks et al. (2008).[8]

The calibration goes as follow. Capacity utilization u_o and potential output y^p are set at 1 in the steady state. This implies that output y_o is normalized to 1 at the steady state. Inventories are assumed away $\nu_o = 0$ in the following simulations by setting $\alpha_{nd} = 0$. Expected output is therefore equal to output $y^e_o = y_o = 1$. The debt to capital ratio λ_o is also normalized to unity.

The simulation considers the case of a polarized distribution of income in which both type of workers consumed their entire incomes, in contrast to capitalists households. The propensity to consume of capitalist c_c is zero, while the propensity to consume of regular worker c_w is 1. Workers pays 10 percent of taxes (net of interest) t^n_w, while asset-holders pays 20 percent of taxes (net of interest) on their real profits. This produces a labour share of income equal to 62 percent in line with historical average. The labour share of income is distributed equally between workers of type I and workers of type II. γ_d, γ_u and the speeds of adjustment in the labour market β_{w1} and β_{w2} are set to 0.09, 0.01, 1.8 and 0.3 respectively. This leads to employment rates, normalized by the labour force of each type of labour, of 90 percent in the type I labour market e_{1o} and 60 percent in the type II labour market e_{2o}. Lastly, prices are normalized to 1 and the inflation rate is zero at the steady states. Nominal wages are relatively more flexible than price with $\beta_{w1} = 1.8$, $\beta_{w2} = 0.3$, $\beta_{wu} = 1.1$, $\beta_{pu} = 1$, while the cost push elements are symmetric $\kappa_w = \kappa_p = 0.5$. The sensitivity of investment to profitability is $i_\rho = 1$, which produces a profit-led demand regime. Given the relative flexibility of nominal wages, the wage-price aggregate demand feedback loop is stable. Monetary policy has a strong stabilizing effect on capital accumulation with $i_r = 0.5$, while the Kaleckian retardation effect is moderate $i_o = 0.3$. Lastly, the Fisher effect is small as $i_\lambda = 0.3$.

Figures 1 to 4 display the adjustment of expected output, the labour share and labour demand of type I following a 5 percent shock on expected output. In figure 2 and 4, v_1 is only the wage share of employment e_1 and Y/x_i is to be viewed as employment functions, not productivity indexes, which as isolated magnitudes are meaningless in the considered joint output situation. The difference between figures 1-2 and figures 3-4 is that noise has been added in the simulations corresponding to the latter set of figures. The amplitude and phase length in figures 1 to 4 are quite satisfying from an empirical point of view with business cycle taking place over a 7 years period of time.[9]

[8] This tool for the numerical analysis of dynamic period models as well as continuous-time ODE systems can be downloaded from the webpage of the CeNDEF.

[9] We have added a *sinh*-nonlinearity to both the fiscal and the monetary policy rule which are thus assumed

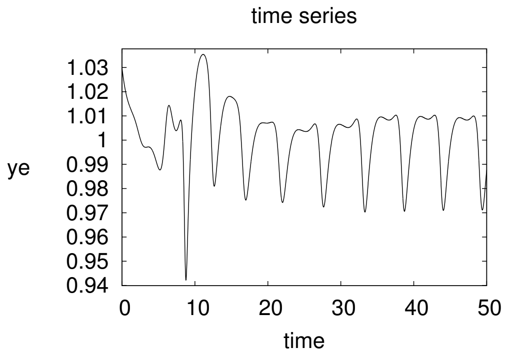

Figure 2: Time series – deterministic case

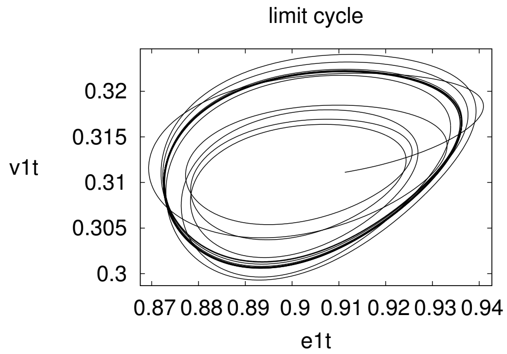

Figure 3: Partial phase plot

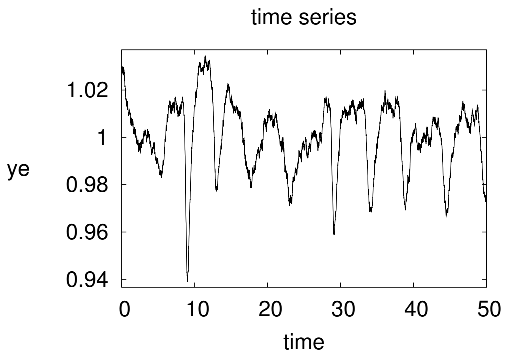

Figure 4: Time series: Adding noise

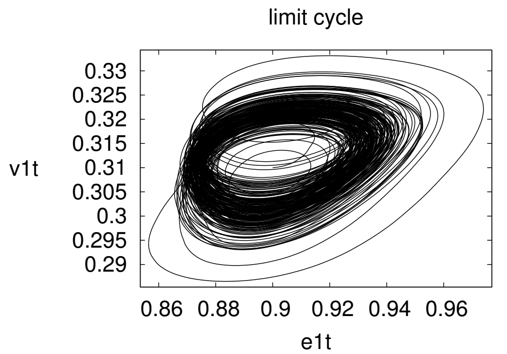

Figure 5: Limit cycle: Adding noise

Further, figure 6 illustrates the fluctuations of the employment rates of workers of type I and II, and the resulting aggregate employment rate e, together with the evolution of y^e. As it can be clearly observed in this figure, our model does not only generate an endogenous employment rate, but this resulting variable closely follows the evolution of aggregate demand, along the Keynesian view on the linkage between the goods and the labor markets.

As next we analyze how the dynamics of the system are affected by ceteris paribus parameter increases. The most relevant of these bifurcation diagrams are illustrated in figures 7-9.

These bifurcation diagrams – which in continuous time should only plot local maxima and minima, but not all points after a transient phase of here 30 years – demonstrate the strong economic viability of the generated cycle mechanism which however becomes larger when the speed of adjustment of the labor force (β_e), i.e., hiring and firing is increased for both e_1, e_2. Increasing the negative impact of the debt to capital ratio i_λ is somewhat stabilizing the cycle,

to be weak around the steady state, but gather force the more the system departs from it. This is the basis of the persistent fluctuations the chosen numerical example creates.

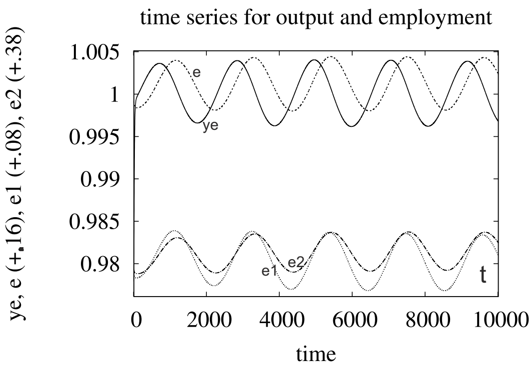

Figure 6: Fluctuations in the total employment rate and its constituent parts

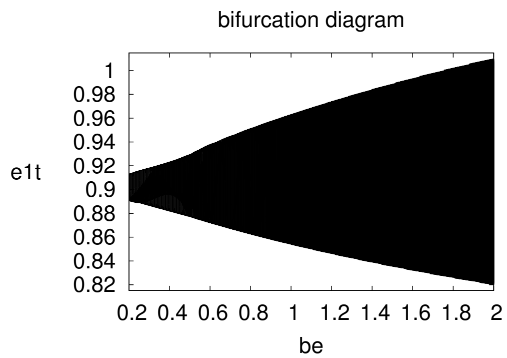

Figure 7: Bifurcation diagram: Varying hiring and firing speeds in the first labor market

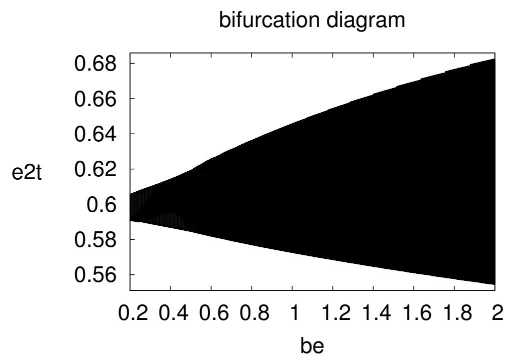

Figure 8: Bifurcation diagram: Varying hiring and firing speeds in the second labor market

while the effect of the inflation rate on the loan rate is not. In all cases it is demonstrated that the dynamics are bounded over large ranges of parameter values.

6 Conclusions

In this paper we set up a dynamic macro model representing detailed goods market dynamics, heterogeneous labor markets, dual and cross-dual adjustment processes, as well as counter-cyclical government policies. The core of the resulting Keynesian macrodynamics is given by credit-financed investment behavior and credit supplies setting the loan rate. As we have shown a variety of mostly destabilizing feedback chains can arise. The framework is constructed in such way so that simplified, lower dimensional versions of the model can be obtained by setting parameters describing specific feedback effects from one sector to another

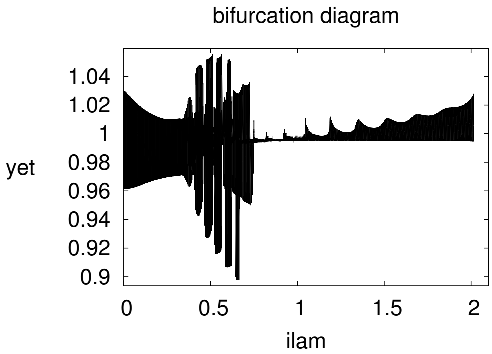

Figure 9: Bifurcation diagram: Varying leverage effects

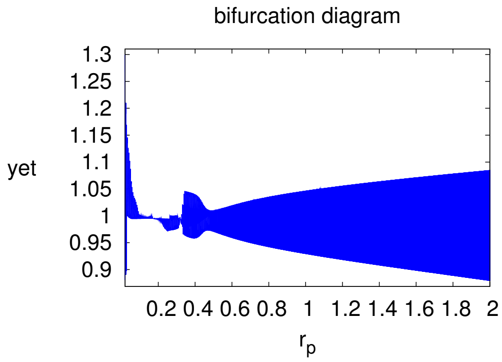

Figure 10: Bifurcation diagram: Varying loan rate acceleration

equal to zero. Starting from such a low dimensional sub-dynamics, we show stability of the full 7D model through a 'cascade of stable matrices' approach if the feedback chains are sufficiently tranquil in their transmission mechanisms. However, local stability is the departure point for the numerical investigation of local explosiveness and the forces that can bound such a behavior. Naturally, when then macro fluctuations arise not only aggregate employment will fluctuate but also – to a different extent – the employment in our two specified segments of the labor market. Our modeling device may help us to understand why through the implementation of "labor market and structural reforms" – originally aimed to reduce labor market frictions, as for example the ones implemented in Germany during the 2000s – have resulted in quite an unpleasant outcome. The type of liberalization policies implemented as labor market reforms has led to an increasing fraction of the labor force being employed in low-income segment of the labor market with the employment being highly volatile.

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